8.251 – Homework 3

B. Zwiebach

Due Thursday, March 1.

1. (5 points) Problem 4.1
2. (10 points) Problem 4.2
3. (15 points) Problem 4.3
4. (10 points) Closed string motion.

We can describe a nonrelativistic closed string fairly accurately by having the string wrapped around a cylinder of large circumference $2\pi R$ on which it is kept taut by the string tension $T_0$. We assume that the string can move on the surface of the cylinder without experiencing any friction. Let $x$ be a coordinate along the circumference of the cylinder: $x \sim x + 2\pi R$ and let $y$ be a coordinate perpendicular to $x$, thus running parallel to the axis of the cylinder. As expected, the general solution for transverse motion is given by

$$y(x,t) = h_+(x-v_0 t) + h_-(x+v_0 t),$$

where $h_+(u)$ and $h_-(v)$ are arbitrary functions of single variables $u$ and $v$ with $-\infty < u, v < \infty$. The string has mass per unit length $\mu_0$, and $v_0 = \sqrt{T_0/\mu_0}$.

(a) State the periodicity condition that must be satisfied by $y(x,t)$ on account of the identification that applies to the $x$ coordinate. Show that the derivatives $h'_+(u)$ and $h'_-(v)$ are, respectively, periodic functions of $u$ and $v$.

(b) Show that one can write

$$h_+(u) = \alpha u + f(u), \quad h_-(v) = \beta v + g(v),$$

where $f$ and $g$ are periodic functions and $\alpha$ and $\beta$ are constants. Give the relation between $\alpha$ and $\beta$ that follows from (a).

(c) Calculate the total momentum carried by the string in the $y$ direction. Is it conserved?

5. (10 points) Problem 5.3
6. (10 points) Problem 5.4
7. (10 points) Problem 5.5

While not assigned, I think all students should know how to solve problem 4.6 and problem 5.1.