Lecture 21 - Topics

- Wrap-up of Closed Strings

Wrapup of Closed Strings

String Coupling Constant

Dimensionless number that sets the strength of the interaction.

Example of coupling constants:

1. Fine structure constant \( \alpha = \frac{e^2}{4\pi\hbar c} \approx \frac{1}{137} \). Derives from interactions between charged particles, magnetic fields. Could imagine particles with \( e = 0 \).

   Action \[ S = \int d^p x (-\frac{1}{4} F^2) - mc \int dS + e \int A_\mu dx^\mu. \] Doesn’t talk about interaction between charged particle and field (both are free).

2. Binding energy of an electron in a H atom. \( E_{\text{binding}} \propto e^4 \) since \( V \propto \frac{e^2}{r} \) in Bohr atom.

   \[ E_{\text{binding}} = \frac{1}{2} \alpha^2 (M_e C^2) \approx \frac{1}{2} \left( \frac{1}{137} \right) 1137(500,000eV) \approx 13eV. \]

   Similar effect in string theory unless gravitons interacting with matter, no gravity!

   \( D \)-dim Newton’s Constant: \[ [G(D)] = L^{D-2} \] (natural units). \( D = 10: [G^{(10)}] = L^8 \Rightarrow G^{(10)} = g^2 (\alpha')^4 \) where \( g \) is the string coupling constant.

   Planck length \( l_p \) related to string length \( l_s \): \( l_p^8 = g^2 l_s^8 \Rightarrow l_p \approx g^{\frac{4}{3}} l_s \)

   Particle View:
String View:

Close string splits into 2 strings. Same coupling constant $g'$ for string interactions.

$g = e^{\phi(x)}$. Field sets value of coupling constant. So $g$ changes with $\phi(x)$. Not a constant, but a dynamical
value. But usually work with constant field.

4D:

\[ G^{(4)} = \frac{G^{(10)}}{V(6)} = \frac{g^2 \alpha'}{V(6)} = \frac{g^2 \alpha'}{(V(6) / \alpha^3)} \]

**Superstrings**

Everyone uses superstrings more than superstrings. Takes a long time to develop all background, so will present intuitively-reasonable results from QFT.

Pauli exclusion principle: multiple fermions cannot occupy the same state.

\( X^\mu(\tau, \sigma) \): Classical variables. Commute \( X^I(\tau, \sigma)X^J(\tau', \sigma') = X^J(\tau', \sigma')X^I(\tau, \sigma) \).

The \( X \)'s behave as boson fields in \((\tau, \sigma)\) space.

In quantum theory, things don’t quite commute. For operators \( A, B \) commutator \([A, B] = A \cdot B - B \cdot A\). \( A \) and \( B \) commute if and only if \([A, B] = 0\).

Define: \( \psi_1^\mu(\tau, \sigma), \psi_2^\mu(\tau, \sigma) \). Classical, anticommuting variables \( \psi_i^\mu(\tau, \sigma) \).

Let \( B_1, B_2 \) be classical anticommuting variables. Then \( B_1 B_2 = -B_2 B_1, B_1 B_1 = -B_1 B_1 \Rightarrow B_1 B_1 = 0 \). Same for all indices.

Set of anticommm. variables \( B_i \):

\[ B_i B_j = -B_j B_i \]

\[ B_i B_i (\text{not summed}) = 0 \]

Example with matrices:

\[ \gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \]
Quantum operators $f_1, f_2$

$$\{f_1, f_2\} = f_1 f_2 + f_2 f_1$$

Operators anticommute if and only if $\{f_1, f_2\} = 0$.

Quantized a scalar field. Got particle state of $n_k$ particles

$$(a_{p_1}^+)^{n_1} (a_{p_2}^+)^{n_2} \ldots (a_{p_k}^+)^{n_k} |\Omega\rangle$$

**Electron Dirac Field**

$$f_{p_1, s_1}^+ f_{p_2, s_2}^+ \cdots f_{p_k, s_k}^+ |\Omega\rangle$$

$p$: momentum, $s$: spin.

Creation operators have nonzero anticommutators with annihilation operators. All $f^+$'s anticommute. This relates to the Pauli exclusion principle and Fermi statistics.

**Action:** $S = S_{\text{Bosonic}} + S_{\text{Fermionic}}$
Action for LC coordinates. Tells you pretty much everything about dynamics of LC variables.

$$S_B = \frac{1}{4\alpha'} \int d\tau d\sigma (\dot{X}^I \dot{X}^I - X''^I X'^I)$$

What Dirac would have written for a fermion in 2D on the worldsheet, but we can a fermion in spacetime.

Dirac equation usually written as $\psi \gamma \partial \psi$

Usually, $A \partial_\sigma A = \frac{1}{2} \partial_\sigma A^2$ but here the $A$'s ($\psi^I$'s) are anticommuting so $A^2 = 0$.

Instead, $A \partial_\sigma A = \partial_\sigma (AA) - (\partial_\sigma A)A$. $A \partial_\sigma A + (\partial_\sigma A)A = \partial_\sigma (AA)$.

Varying $\delta \psi^I_1, \delta \psi^I_2$

$$\delta S_F = \frac{1}{\pi} \int d\tau d\sigma [\delta \psi^I_1 (\partial_\tau + \partial_\sigma) \psi^I_1 + \delta \psi^I_2 (\partial_\tau - \partial_\sigma) \psi^I_2] + \frac{1}{2\pi} \int d\tau (\delta \psi^I_1 \delta \psi^I_2 - \psi^I_2 \delta \psi^I_1)_{\tau = 0}$$

$$\begin{align*}
(\partial_\tau + \partial_\sigma) \psi^I_1 &= 0 \\
(\partial_\tau - \partial_\sigma) \psi^I_2 &= 0
\end{align*}$$

BC: $\psi^I_1 (\tau, \sigma_*) \delta \psi^I_1 (\tau, \sigma_*) - \psi^I_2 (\tau, \sigma_*) \delta \psi^I_2 (\tau, \sigma_*) = 0$

$$\begin{align*}
\psi^I_1 &= \Psi^I_1 (\tau - \sigma) \\
\psi^I_2 &= \Psi^I_2 (\tau + \sigma)
\end{align*}$$

$$\psi^I_1 (\tau, \sigma_*) = \pm \psi^I_2 (\tau, \sigma_*)$$
Satisfies BC without too much violence.

\[ \delta \psi'_1(\tau, \sigma) = \pm \delta \psi'_2(\tau, \sigma) \]

\( \sigma = 0: \)

\[ \psi'_1(\tau, 0) = \pm \psi'_2(\tau, 0) \]

Choose sign to be positive since action doesn't care, but then can't change sign of field. We have two choices:

\[ \psi'_1(\tau, \pi) = + \psi'_2(\tau, \pi) \]

or

\[ \psi'_1(\tau, \pi) = - \psi'_2(\tau, \pi) \]

\[ \Psi'_1(\tau, \sigma) = \begin{cases} 
\psi'_1(\tau, \sigma) & \sigma \in [0, \pi] \\
\psi'_2(\tau, -\sigma) & \sigma \in [-\pi, 0] 
\end{cases} \]

Continuous field over \([-\pi, \pi]\) since \(\psi'_1(\tau, 0) = \psi'_2(\tau, 0)\)

\[ \Psi'(\tau, \pi) = \psi'_1(\tau, \pi) = \pm \psi'_2(\tau, \pi) = \pm \Psi(\tau, -\pi) \]

\( \Psi \) fermion field is periodic if choose positive sign, antiperiodic if choose negative sign.

Suppose choose periodic (Rammond sector)

\[ \Psi(\tau, \sigma) = \sum_{n \in \mathbb{Z}} d'_n e^{-in(\tau - \sigma)} \]

Suppose choose antiperiodic (Neveu-Schawrz sector)

\[ \Psi'(\tau, \sigma) = \sum_{r \in \mathbb{Z} + \frac{1}{2}} b'_r e^{-ir(\tau - \sigma)} \]