1. The reaction $p + n \leftrightarrow d + Q$, where $Q = 2.2\text{MeV}$, is an important one in the history of the universe, occurring when its temperature and density were much higher than at present. As time goes by the universe gets cooler and less dense and a larger fraction of the available protons and neutrons combine. Suppose that when the number densities of neutrons and deuterons are equal, $n_n = n_d$, the temperature is $T = 8.62 \times 10^8\text{K}$. Find the number density of protons at this time. The deuteron has $g_d = 3$.

(2) In class we derived the jump conditions for adiabatic shocks,

$$
\rho_1 v_1 = \rho_2 v_2,
$$

$$
P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2,
$$

and

$$
\frac{1}{2}v_1^2 + \frac{\gamma}{\gamma - 1} P_1/\rho_1 = \frac{1}{2}v_2^2 + \frac{\gamma}{\gamma - 1} P_2/\rho_2.
$$

Derive the expressions for $\rho_2/\rho_1$, $P_2/P_1$ and $T_2/T_1$ (in terms of adiabatic index and Mach number) presented without proof in class.

(3) Consider a point explosion with total energy $E_{\text{tot}}$ at the center of an infinite star whose density is given by $\rho = \rho_0(r_0/r)^n$.

a) Suppose that explosion generates a stron, self-similar, Sedov-type shock, such that the temperature immediately behind the shock varies inversely as the first power of the time since the explosion. Find the numerical value of the exponent $n$.

b) Consider a point explosion with total energy $E_{\text{tot}}$ at the center of an infinite star whose density is given by $\rho = \rho_0(r_0/r)^{32/11}$. This isn’t a bad approximation for the envelope of a red giant. Use dimensional arguments to determine the power law dependence of radius and velocity with time.

(4) Consider a cylindrically symmetric explosion which releases an energy per unit length $\lambda$ along the $z$-axis of a cylindrical coordinate system into a medium of uniform density $\rho$. Using dimensional arguments, determine the power law dependence of radius and velocity with time.