1. (a) Using linear response theory, derive the following expression for the magnetic susceptibility $\chi = \partial M_z / \partial H_z$.

$$\chi = \lim_{q \to 0} \int \frac{d\omega}{2\pi} \left( S_z(q, \omega) S_z(-q, -\omega) \right) \frac{\left( 1 - e^{-\frac{\omega}{kT}} \right)}{\omega}$$

(b) Provided that the total magnetization $M_z = \sum_i S_{iz}$ commutes with the Hamiltonian, we can start from the expression $F = -kT \ln \text{Tr} \{ e^{-\beta(H-M_z H_z)} \}$ and take derivatives with respect to $H_z$ to derive the simpler expression

$$\chi = \frac{1}{kT} \langle M_z^2 \rangle$$

Show that this is consistent with the more general expression obtained in 2(a).

[Hint: in this special case $\lim_{q \to 0} \langle |S_z(q, \omega)|^2 \rangle \sim \delta(\omega).]$