Problem 1: Triangle Group

Consider symmetries of the equilateral triangle, see Fig. 1. It is carried into itself by a clockwise rotation through 120 ($R_+$), and by a counterclockwise rotation through 120 ($R_-$), by flipping it about the vertical axis $a$ ($R_a$) and axis $b$ ($R_b$) and $c$ ($R_c$). Construct a multiplication table for the triangle group, filling the blanks in Tab. 2. In row $i$, column $j$ put the product $R_iR_j$. Is this an Abelian group? How can you tell, just by looking at the table?
Figure 1: Equilateral triangle.

Figure 2: Multiplication table for triangle group.
The group is not Abelian; the multiplication table is not symmetrical across the main diagonal (for example, $R_+R_a = R_b$, but $R_aR_+ = R_c$).

Figure 3: Multiplication table for triangle group.
Problem 2: Isosping - dynamic implications

Consider three nucleon-nucleon scattering processes

(a) $p + p \rightarrow d + \pi^+$
(b) $p + n \rightarrow d + \pi^0$
(c) $n + n \rightarrow d + \pi^-$

Figure 4: Nucleon-nucleon scattering processes.

The deuteron has isospin $I=0$ and the pion $I=1$. Isospin is conserved in the scattering process. Cross-sections go like the absolute square of the amplitude. What is the ratio of cross sections, $\sigma_a : \sigma_b : \sigma_c$?

Since the deuteron carries $I=0$, the isospin states on the right are $|1,1\rangle$, $|1,0\rangle$, and $|1,-1\rangle$, respectively, whereas those on the left are $p = |1,1\rangle$, $nn = |1,-1\rangle$, and $pn = (|1,0\rangle + |0,0\rangle)^*$ only the $I=1$ combination contributes (since the final state in each case is pure $I=1$, and isospin is conserved), so the scattering amplitudes are in the ratio

$$\mathcal{M}_a : \mathcal{M}_b : \mathcal{M}_c = 1 : \left(\frac{1}{\sqrt{2}}\right) : 1$$

(4.42)

As we shall see, the cross section $\sigma_a$ goes like the absolute square of the amplitude, thus

$$\sigma_a : \sigma_b : \sigma_c = 2 : 1 : 2$$

(4.43)

Process (c) would be hard to set up in the laboratory, but (a) and (b) have been measured, and (when corrections are made for electromagnetic effects) they are found to be in the predicted ratio [7].

Figure 5: Answer.

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