Soft-Collinear Effective Theory (SCET)

For this part we'll switch sign convention for $g$

\[
\bar{e} \gamma^\mu e = i g T^A \gamma^\mu \text{ to agree with literature}
\]

Outline

Class 1: Intro, Degrees of Freedom, Scales, Expansion of Spinors, Propagators, Power Counting see ②, ③

Class 2: Construction of Currents, Lagrangian, Multipole Expansion, Labels, Grid in detail see ②, ③, ⑤ (not in notes)

SCETⅡ

Class 3: Lagrangian, Gauge Symmetry, ⑤, ⑥, ⑩ Reparameterization Invariance (RPI)

Class 4: More RPI, Ultrasoft-Collinear Fact., Hard-Collinear Factorization, IR divs., Matching, Running see ④, ①, ②, ③

Class 5: DIS see ⑧ Soft-Collinear Interactions ④

Class 6: SCET ④, ⑦, ⑩ Power Counting Formulas ⑤

ag. $\gamma^\nu q \rightarrow \pi^0$ ②, eq. $B \rightarrow D\pi$ ④

ag. $B \rightarrow X_{5\gamma}$. Define a Jet ④

(Jets in $e^+e^-$, see ⑪)

Refs I used

0 hep-ph/0005275
1 hep-ph/0001336
2 hep-ph/0107001
3 hep-ph/0109045
4 hep-ph/0209289
5 hep-ph/0204229
6 hep-ph/0205289
7 hep-ph/0303156
8 hep-ph/0202089
9 hep-ph/0107802
10 hep-ph/0605001
11 hep-ph/0212255
12 hep-ph/0603066
We work an EFT for energetic hadrons: $E_H \approx Q \gg \Lambda_{QCD}$

Why? Many processes have large regions of phase space
when the hadrons are energetic, $E_H \gg M_H$

- $B$-decays: $B \to \pi \nu\bar{\nu}$, $B \to K^* \gamma$, $B \to \pi \pi$, $B \to X \nu \bar{\nu}$
  - $B \to X(\gamma)$, $B \to D^* \pi$, ...
  - $M_B = 5.279$ GeV $\gg \Lambda_{QCD}$

Hence scattering:

- $e^- e^- \to e^- X (Drell-Yan)$
- $p\bar{p} \to X e^+ e^-$

- $\gamma^* \gamma \to \pi^0$
- $\gamma^* p \to \gamma^{(*)} p'$ (Deeply Virtual Compton Scattering)

- Need to separate perturbative, $ds(Q)$ and non-perturbative
  "$ds(\Lambda_{QCD})"$ effects $\rightarrow$ factorization

What are the low energy degrees of freedom?

- $B \to D \pi$

  In $B$-rest frame:

  $p_\pi = (2.310, 0, 0, -2.304)$ GeV

  $Q \approx \Lambda$, $n^\mu = (1, 0, 0, -1)$, $n^\nu = 0$; light-like

  In $0, 1, 2, 3$ basis
Use light-cone coordinates: $n^2 = 0$, $\bar{n}^2 = 0$, $n \cdot \bar{n} = 2$.

Vectors $P^\mu = \frac{n^\mu}{2} n \cdot p + \frac{\bar{n}^\mu}{2} n \cdot p + P_\perp^\mu$.

Metric $g^{\mu \nu} = \frac{n^\mu n^\nu}{2} + \frac{\bar{n}^\mu \bar{n}^\nu}{2} + g_\perp^{\mu \nu}$.

Orthogonal $n^\mu, \bar{n}^\mu$.

Define $p^+ \equiv n \cdot p$, $p^- \equiv \bar{n} \cdot p$.

Since $n^2 = 0$ we needed to define complementary vector $\bar{n}$.

Choice $n^\mu = (1, 0, 0, -1)$, $\bar{n}^\mu = (1, 0, 0, 1)$ is possible, but other choices also work, e.g., $n^\mu = (1, 0, 0, -1)$, $\bar{n}^\mu = (3, 2, 2, 1)$.

(more on this later)

In $B \to D\pi$ the $B, D$ are soft, $E_H \sim M_H$ and we can use HQET for their constituents, i.e., quarks & gluons with $p^\mu \sim \Lambda$.

But pion is "collinear", $E_H \gg M_H$.

In rest frame $\pi$ has quark & gluon constituents.

Boosting for $B \to D\pi$ gives constituents $p^\mu \sim (\Lambda, \Lambda, \Lambda)$.

Fluctuations around $(0, 0, 0) = p^\mu_0$.

Note: Boost in direction orthogonal to $\perp$ direction changes $p^+, p^-$ multiplicatively $p^+ \to \alpha p^+$, $p^- \to \frac{1}{\alpha} p^-$. 
Generically,

\[(p^+, p^-, p^z) \sim Q (\lambda^2, 1, \lambda) \text{ is collinear}\]

where \(\lambda \ll 1\) is small parameter. (above eq. \(\lambda = \frac{A}{Q}\))

What makes this EFT different?

- usually we separate scales \(M_1 \gg M_2\) and have

\[\sum_{i=1}^{\infty} C_i (\mu_1, M_1) U_i (\mu_2, M_2)\]

\[\text{short distance} \quad \text{Wilson Coeffs} \quad \text{long distance} \quad \text{operators}\]

\(\Lambda\) in HQET

the B-meson

\[M_B \gg \Lambda\]

\[p_\alpha \sim M_B\]

\[p_\mu \sim \Lambda\]

\[p_\alpha \sim m_B^2\]

\[p_\mu \sim \Lambda^2\]

\[\text{picture momenta}\]

\[p^+ \sim \Lambda^2\]

\[p^- \sim \Lambda^2\]

\[p^z \sim \Lambda^2\]

\[p_\alpha \sim \Lambda^2\]

\[p_\mu \sim \Lambda^2\]

\[\text{well separated in all components}\]

- now we have overlap between perturbative & non-perturbative momenta in \(p^-\) component

for collinear pion

\[E_{\pi} \sim M_B\]

\[p_c \sim (\frac{\Lambda^2}{M_B}, M_B, \Lambda)\]

\[\text{overlap in } p^-\text{, but } p_c \ll p_\alpha \text{ still}\]
2. inclusive decay $B \rightarrow X_s \tau^+$ from $b \rightarrow s \tau^+$

... $\geq 1$ hadron, summed over

... in general $E \gamma = \frac{M_B^2 - M_{X_s}^2}{2m_B} \epsilon \left[ 0, \frac{M_B^2 - M_{X_s}^2}{2m_B} \right]$

... for $M_X \in [m_b, M_{X_s}]$

For $M_X \sim M_B^2$

... standard OPE

... just like we

... X has hadrons in all directions

... did for $B \rightarrow X_c \ell^+$

For $M_X^2 \sim \Lambda^2$

... exclusive decay

... (not inclusive)

For $M_X^2 \sim M_B^2$

... jet of hadrons in X

... $\left( p^+, p^-, p_z \right) \sim \left( \Lambda, Q, \sqrt{sQ} \right) \sim Q \left( \lambda^2, 1, \Lambda \right)$

... constituents

... collinear again

... this time $\lambda = \frac{\sqrt{sQ}}{Q} \ll 1$
Infrared Degrees of Freedom have \( p^2 \lesssim Q^2 x^2 \)

<table>
<thead>
<tr>
<th>modes</th>
<th>( p^x = (+, - , \pm) )</th>
<th>( p^z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>collinear</td>
<td>( Q (x^2, 1, 1) )</td>
<td>( Q^2 x^2 )</td>
</tr>
<tr>
<td>soft</td>
<td>( Q (x, x, 1) )</td>
<td>( Q^2 x )</td>
</tr>
<tr>
<td>ultrarelativistic (soft)</td>
<td>( Q (x^2, x, x) )</td>
<td>( Q^2 x^4 )</td>
</tr>
</tbody>
</table>

Off-shell modes have \( p^2 \gg Q^2 x^2 \) and are integrated out into Wilson coefficients \( C(\mu) \)

\[ p^{\mu} \sim Q (1, 1, 1) \]

Two useful cases

**SCET**

\[ \lambda = \frac{\Lambda}{Q} \]

\[ \sum [ \text{collinear} \; p_c^z \sim Q \Lambda \]

\[ \text{soft} \; p_s^z \sim \Lambda \]

**SCET**

\[ \lambda = \frac{\Lambda}{Q} \]

\[ \sum [ \text{collinear} \; p_c^z \sim \Lambda^2 \]

\[ \text{soft} \; p_s^z \sim \Lambda^2 \]

Examples

\( B \to XsY \)

*DIS*, ...

\( B \to D \pi, \)

\( \delta \to \pi + \pi, \)

The theory \( \text{SCET} \) can be derived from \( \text{SCET} \)

so we'll study \( \text{I} \) first

Factorization: \( \sum_i C_i O_i \) becomes continuous

\[ \int d\xi \; C(\xi) O(\xi) \]

since \( p^- \) were same size
Collinear Spinors \( U_n \) labelled by direction \( n \) (recall HAE compression).

Massless \( O(4) \) spinors

\[
U(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} U \\ \sigma^3 U \end{pmatrix}, \quad \gamma(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} p \cdot \sigma \\ p^0 \end{pmatrix}
\]

Let \( n^\mu = (1, 0, 0, 1) \) and expand, \( \bar{n} \cdot p = p^0 + p^3 = \frac{\sigma^3}{2} + \frac{q^2}{2} \), \( \bar{n} = (1, 0, 0, -1) \) \( n \cdot p \ll q, \ p \ll q \), \( \frac{\sigma \cdot \sigma}{p^0} = \sigma^3 \)

\[
U_n = \frac{1}{\sqrt{2}} \begin{pmatrix} U \\ \sigma^3 U \end{pmatrix} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \quad \text{particles}
\]

\[
\bar{U}_n = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma^3 \gamma \\ \gamma \end{pmatrix} = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \quad \text{antiparticles}
\]

\[
\chi = \begin{pmatrix} 1 - \sigma^3 \\ \sigma^3 - 1 \end{pmatrix} \quad \chi U_n = \chi \bar{U}_n = 0
\]

\[
\frac{\chi \bar{U}}{4} = \frac{1}{2} \begin{pmatrix} 1 & \sigma^3 \\ \sigma^3 & 1 \end{pmatrix} \quad \frac{\chi \bar{U}}{4} U_n = U_n, \quad \frac{\chi \bar{U}}{4} \bar{U}_n = \bar{U}_n
\]

Projection Operator \( \frac{1}{4} = \frac{\chi \bar{U}}{4} + \frac{U \bar{U}}{4} \)

Field \( \gamma^0 \sigma \equiv \gamma_n + \gamma_\bar{n} \)

We'll integrate out "small" component \( \gamma_\bar{n} \)
Collinear Propagators

\[ p^2 + i\epsilon = \frac{i}{\bar{n} \cdot p} n \cdot p + \frac{p^2}{n \cdot p} + i\epsilon \]

\[ \sim n^2 + 2n \lambda \quad \text{Same size} \]

Fermions

\[ \frac{\bar{\psi} \gamma^\mu \psi}{p^2 + i\epsilon} = \frac{i}{2} \frac{\bar{n} \cdot p}{p^2 + i\epsilon} \]

\[ = \frac{i}{2} \frac{1}{\bar{n} \cdot p + \frac{p^2}{\bar{n} \cdot p} + i\epsilon \text{ sign}(\bar{n} \cdot p)} + \ldots \]

\[ \uparrow \text{from } T \in \gamma_\lambda(x), \gamma_\mu(0) \}

Gluons

\[ \frac{-i g F_{\mu\nu}}{p^2 + i\epsilon} \quad \text{stays same as QCD} \quad g^{\mu\nu} a^\lambda \]

\[ \uparrow \quad (\text{as Feynman Gauge}) \]

Power counting for collinear fields

\[ \lambda = \int d^4 x \; T_{\mu\nu} \frac{\bar{\psi} \gamma^\mu \psi}{2} \left[ i n \cdot \gamma + \ldots \right] \eta_0 \]

\[ T_{\mu\nu} \lambda^\mu \lambda^\nu \lambda^2 = \lambda^{2t-2} \]

Set \( \lambda \sim A_0 \) ie. normalize kinetic term so no \( A_0 \)'s

then

\[ \eta_0 \sim \lambda \]

For gluons find \[ A_0^\mu = (A^+_0, A^-_0, A^{\perp}_0) \sim (\lambda^0, i, \lambda) \]

just like collinear momenta

ie have \[ p^\mu + A_0^\mu = i D^\mu \quad \text{homogeneous covariant derivative} \]