Lecture 3 Game Plan

- Nash equilibrium
  - ... in pure strategies & mixed strategies
  - ... how NE play arises from rationality
  - ... how NE play can arise from evolution
Nash Equilibrium

- **Nash Equilibrium:**
  - A set of strategies, one for each player, such that each player’s strategy is a best response to others’ strategies.

- **Best Response:**
  - The strategy that maximizes my payoff given others’ strategies.

- **Everybody is playing a best response**
  - No incentive to unilaterally change my strategy.

Slide courtesy of Mike Shor, Vanderbilt University.
Some Prototypical Games

- Prisoners’ Dilemma
- price war
Example: SUV Price Wars

“General Motors Corp. and Ford Motor Co. slapped larger incentives on popular sport-utility vehicles, escalating a discounting war in the light-truck category ... Ford added a $500 rebate on SUVs, boosting cash discounts to $2,500. The Dearborn, Mich., auto maker followed GM, which earlier in the week began offering $2,500 rebates on many of its SUVs.”

## SUV Price Wars: The Game

<table>
<thead>
<tr>
<th></th>
<th>Discount</th>
<th>Don’t</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GM</strong></td>
<td>![Sad] ![Sad] ![Smiley] ![Sad]</td>
<td>![Sad] ![Smiley] ![Sad] ![Sad]</td>
</tr>
<tr>
<td><strong>Ford</strong></td>
<td>![Sad] ![Smiley] ![Sad] ![Cry]</td>
<td>![Sad] ![Smiley] ![Sad] ![Sad]</td>
</tr>
</tbody>
</table>

*Note: Smiley face represents a positive outcome, Sad face represents a negative outcome, and Cry face represents a very negative outcome.*
SUV Price Wars: Outcome

- Each firm has a unilateral incentive to discount but neither achieves a pricing advantage.
“Red Queen Effect”

“It takes all the running you can do to keep in the same place” – Red Queen to Alice

From Carroll, Lewis. Alice’s Adventures in Wonderland.
Prisoners’ Dilemma

SUV Price War is a “prisoners’ dilemma” game:

1. Both firms prefer to Discount regardless of what other does. (Discount is a dominant strategy.)
2. BUT both firms are worse off when they both Discount than if they both Don’t.
Prisoners’ Dilemma Game

Key features:
- Both players have a dominant strategy to Confess
- BUT both players better off if they both Don’t
Prisoners’ Dilemma Game

<table>
<thead>
<tr>
<th>Prisoner 1</th>
<th>Confess</th>
<th>Don’t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confess</td>
<td><img src="image" alt="Confess, Confess" /></td>
<td><img src="image" alt="Don’t, Confess" /></td>
</tr>
<tr>
<td>Don’t</td>
<td><img src="image" alt="Don’t, Confess" /></td>
<td><img src="image" alt="Don’t, Don’t" /></td>
</tr>
</tbody>
</table>

- If Prisoner 1 confesses, Prisoner 2’s best choice is to confess, leading to both being sad. (Both are in the top-left cell)
- If Prisoner 1 doesn’t confess, Prisoner 2’s best choice is to confess, leading to Prisoner 1 being sad and Prisoner 2 being happy. (Both are in the bottom-left cell)

The optimal strategy is for neither to confess, but they are locked into confessing. This is a classic example of a coordination problem.
Reaction Curves in Prisoners’ Dilemma

Prisoner 2’s prob. of Confess

0% 50% 100%

Prisoner 1’s prob. of Confess

0% 50% 100%
Evolution in Prisoners’ Dilemma (One Population)

- Row and Col players are drawn from the same population
- Those who Confess get higher payoff, so Confess dominates the population
Some Prototypical Games

- Prisoners’ Dilemma
- Loyal Servant
- price war
- defensive innovation
Soft & Chewy Cookies

A cookie store is a bad idea. Besides, the market research reports say America likes crispy cookies, not soft and chewy cookies like yours.

-Response to Debbi Fields' idea of starting Mrs. Fields' Cookies, 1977
Soft & Chewy Cookies

- Supermarket cookies tend to be crispy, not chewy
- Duncan Hines (owned by P&G) entered with a chewy cookie [1984]
- How did Nabisco and Keebler respond?
Soft & Chewy Wars

- Nabisco and Keebler rolled out their own soft and chewy varieties:
  - Keebler Soft Batch
  - Nabisco Chips Ahoy! Chewy
Soft & Chewy Settlement

- Duncan Hines brings patent-infringement suit alleging industrial espionage by Keebler, Nabisco and Frito-Lay [1984]
- Companies agree to pay P&G $125 million, then the most ever reported to settle a patent lawsuit [1989]
Soft & Chewy Retreat

- P&G sells Duncan Hines to Aurora [1997]

“This agreement is a win-win. Consumers will still be able to buy great Duncan Hines products, now through Aurora Foods, and we can focus on the strategic opportunities we've established for our food and beverage business."

-Steve Donovan, P&G VP, food and beverages.

Defensive Innovation

- A monopolist’s incentive to innovate increases as it faces innovative entrants.
- Consider case of *product variety*
  - consumers represented as points in a square
  - they buy whichever product is closest
Product Variety Game

Without any entry, the monopolist doesn’t gain from introducing a new product
  • To keep things simple, we suppose price is fixed
Now suppose an entrant comes in with a new variety. Now “landing on the entrant” keeps some customers

- Incentive to introduce new variety if entry
- Will the other firm enter anyway?
# Defensive Innovation: Summary So Far

<table>
<thead>
<tr>
<th></th>
<th>Innovate</th>
<th>Don’t</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Innovate</strong></td>
<td><img src="??" alt="Smiley" /></td>
<td><img src="??" alt="Smiley" /></td>
</tr>
<tr>
<td><strong>Entrant</strong></td>
<td><img src="??" alt="Smiley" /></td>
<td><img src="??" alt="Sad" /></td>
</tr>
<tr>
<td><strong>Don’t</strong></td>
<td><img src="??" alt="Sad" /></td>
<td><img src="??" alt="Smiley" /></td>
</tr>
</tbody>
</table>

- **Monopolist**
- **Innovate**: ??
- **Don’t**: ??

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Case I: Entrant Wants to Innovate Anyway

Monopolist

Entrant

Innovate

Don’t

Innovate

Don’t
What Can Conceivably Happen?

1. ... if both Monopolist and Entrant are rational (only)
   - (Innovate, Innovate) or (Innovate, Don’t)
     - latter requires monopolist to mistakenly believe that entrant will not innovate

2. ... if rationality is common knowledge?
   - (Innovate, Innovate) only since monopolist knows entrant is rational!
Loyal Servant Game*

Key features:
- One player (Master) has dominant strategy
- Other player (Servant) wants to do the same thing as Master

*Not discussed in textbook
Loyal Servant Game

Servant

Safe Route  Dangerous

Safe Route

Master

Dangerous
Reaction Curves in Loyal Servant Game

**Servant’s prob. of Safe**

- 0%
- 50%
- 100%

**Master’s prob. of Safe**

- 0%
- 50%
- 100%
Evolution in Loyal Servant Game (Two Populations)

Evolution leads to Nash eqm (Safe, Safe)
Rationalizable Strategies

- Strategies are “rationalizable” if they could *conceivably* be played when
  1. players are rational and
  2. rationality is common knowledge

- Suppose each player has a unique rationalizable strategy. Then these strategies form a Nash equilibrium.
How to Find Rationalizable Strategies

- If a strategy is strictly dominated for some player, eliminate it.
- Repeat, eliminating any strictly dominated strategies in reduced game.
- All strategies that remain when you are finished are rationalizable.
Example: Tourists & Natives

- Two bars (bar 1, bar 2) compete
- Can charge price of $2, $4, or $5
- 6000 tourists pick a bar randomly
- 4000 natives select the lowest price bar

- Example 1: Both charge $2
  - each gets 5,000 customers
- Example 2: Bar 1 charges $4, Bar 2 charges $5
  - Bar 1 gets 3000 + 4000 = 7,000 customers
  - Bar 2 gets 3000 customers

Slide courtesy of Mike Shor, Vanderbilt University.
# Tourists & Natives

<table>
<thead>
<tr>
<th>Bar 1</th>
<th>Bar 2</th>
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</thead>
<tbody>
<tr>
<td>$2</td>
<td>$2</td>
</tr>
<tr>
<td>10, 10</td>
<td>14, 12</td>
</tr>
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<td>20, 20</td>
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<tr>
<td>15, 14</td>
<td>15, 28</td>
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<tr>
<td>$4</td>
<td>$4</td>
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<td>14, 15</td>
<td></td>
</tr>
<tr>
<td>28, 15</td>
<td></td>
</tr>
<tr>
<td>$5</td>
<td>$5</td>
</tr>
<tr>
<td>25, 25</td>
<td></td>
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</tbody>
</table>

*in thousands of dollars*
Successive Elimination of Dominated Strategies

<table>
<thead>
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<th></th>
<th>Bar 1</th>
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<th>Bar 2</th>
<th></th>
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<tbody>
<tr>
<td>$2$</td>
<td>10, 10</td>
<td>14, 12</td>
<td>14, 15</td>
<td></td>
</tr>
<tr>
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<td>12, 14</td>
<td>20, 20</td>
<td>28, 15</td>
<td></td>
</tr>
<tr>
<td>$5$</td>
<td>15, 14</td>
<td>15, 28</td>
<td>25, 25</td>
<td></td>
</tr>
</tbody>
</table>

Bar 1

<table>
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<th>Bar 1</th>
<th></th>
<th>Bar 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$4$</td>
<td>20, 20</td>
<td>28, 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5$</td>
<td>15, 28</td>
<td>25, 25</td>
<td></td>
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</table>

Slide courtesy of Mike Shor, Vanderbilt University.
Some Prototypical Games

- Prisoner’s Dilemma
- Loyal Servant
- **Hunter and Hunted**
- price war
- defensive innovation
- audits, bluffing
Online Game #4

Monitoring Game
In-Class Game

Bluffing Game
Bluffing in Poker: Set-Up

Player A’s hand prior to getting 5th card

- Player A will be drawing on an inside straight flush
- Player A will have the best hand if:
  - flush (another club: 9 cards total) or
  - straight (any 2 or 7: additional 6 cards)
Winning Cards

(Road sign: “Deuce of Clubs Ave.”)

(Sign: “City of Show Low, AZ”)
Bluffing Game: Rules

- Each player has put $100 into the pot
- After receiving the fifth card, player A will either **Raise $100** or **Not**
- If Raise, Player B then either **Calls** (adds $100 more) or **Folds** (automatically losing $100 already in pot)
- Player A wins the pot if *either* A gets winning card *or* B folds
Bluffing Game: Rules

- **Good Card**
  - Raise
  - Not
    - Call: 200, -200
    - Fold: 100, -100

- **Bad Card**
  - Raise
  - Not
    - Call: -200, 200
    - Fold: 100, -100

*successful bluff!*
Play Bluffing Game!

- Pair up with a neighbor.
- Player A will be given a playing card.
- After that, communication allowed.
  - Players A,B may say (or show) anything they want to each other.
Defensive Innovation Case II: Entrant Wouldn’t Innovate

Monopolist

Entrant

Innovate

Don’t
Hunter and Hunted Game*

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Left</strong></td>
<td>(2,-2)</td>
<td>(-2,0)</td>
</tr>
<tr>
<td><strong>Right</strong></td>
<td>(0,2)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

**Key features:**
- Hunter wants to “catch”; Hunted wants to “avoid”

*Called “Attack & Defend” game in textbook*
Reaction Curves in Hunter and Hunted Game

Hunted’s prob. of Right

Hunter’s prob. of Right
Evolving populations may *cycle around* or *fall into* the mixed strategy equilibrium, depending on details.
Side-Blotched Lizard

Orange throat, "dominating"

Blue throat, "monogamous"

Yellow throat, "sneaky"
Summary

- Recognize dominant strategies
  - Prisoners’ Dilemma
- Take others’ (ir)rationality into account
  - Loyal Servant Game
- Mixing can be the right way to play
  - Hunter and Hunted Game
- Next time: more on evolution and introducing sequential moves
Online Game #3
(Entrant Game)

- Play Online Game #3 prior to midnight before next lecture.

- Note: We are *not* playing the games in their numerical order!!