15.407 Sample Mid-Term Examination – Fall 2008

Please check to be certain that your copy of this examination contains 18 pages (including this one). Write your name and MIT ID number on every page. You are allowed one 8½”×11” sheet of notes and one non-programmable non-PDA calculator. Unlike the group projects, you are to answer these examination questions without consulting anyone. No scratch paper is allowed—do all your work on these examination pages.

For simplicity, assume that the coupon payments for all coupon bonds occur annually, i.e., once a year, not semi-annually as is the convention in practice.

You have eighty (80) minutes to complete this examination. Credit for each question is proportional to the amount of time you ought to spend on it. Therefore, do not agonize over a 10-point question without having tackled a 40-point question. Good luck!

### Some Useful Formulas

\[
\text{NPV} = \sum_{k=1}^{n} \frac{C}{(1+r)^k} = \frac{C}{r} \left[ 1 - \frac{1}{(1+r)^n} \right] \quad \text{(Annuity)} \quad (1)
\]

\[
\text{NPV} = \sum_{k=1}^{\infty} \frac{C}{(1+r)^k} = \frac{C}{r} \quad \text{(Perpetuity)} \quad (2)
\]

\[
\text{Price} = \sum_{k=1}^{\infty} \frac{D}{(1+r)^k} = \frac{D}{r} \quad \text{(DDM)} \quad (3)
\]

\[
\text{Price} = \sum_{k=1}^{\infty} \frac{D(1+g)^{k-1}}{(1+r)^k} = \frac{D}{r-g} \quad \text{(DDM with growth)} \quad (4)
\]
15.407 Mid-Term Examination 2008 Grade Sheet

1. ______ / 10
2. ______ / 10
3. ______ / 20
4. ______ / 30
5. ______ / 20
6. ______ / 30
Total ______ / 120
Question 1 (10 points):

If the annual interest rate is 10 percent, how long would you have to wait before a $17,500 investment at least doubles in value? Give the minimum number of years required.
Question 2 (10 points):

Your car dealer offers you a loan for part of the purchase price of a new car, citing an annual percentage rate (APR) of 8.5%. What is the effective annual rate of such a loan (recall that an auto loan typically requires monthly payments)?
Question 3 (20 points):

Company ABC has just paid a dividend of 50 cents per share. Because of its growth potential, its dividend is forecasted to grow at a rate of 7 percent per year indefinitely. If the company’s appropriate cost of capital (given its risk) is 11 percent, what was ABC’s share price immediately before it paid its 50 cent dividend, i.e., the stock price right before the ex-dividend date?
Extra Space For Question 3:
Question 4 (30 points):

The current market price of a two-year 25 percent coupon bond with a $1,000 face value is $1,219.71 (recall that such a bond pays coupons of $250 at the end of years 1 and 2, and the principal of $1,000 at the end of year 2). The current market price of a one-year pure discount bond with a $50 face value is $44.64.

(a) (15 points) What must the price of a two-year pure discount bond with a $2,500 face value be in order to avoid arbitrage?
Question 4 Continued:

(b) (15 points) Suppose the price of the two-year pure discount bond with a $2,500 face value is only $1,900. Is there an arbitrage opportunity? Is yes, how would you structure a trade that has zero cash-flow in years 1 and 2 and a positive cash-flow only in year 0 (i.e. now).
Question 5 (20 points):

Your friend is celebrating her 35th birthday today and wants to start saving for her anticipated retirement at age 65 (she will retire on her 65th birthday). She would like to be able to withdraw $80,000 from her savings account on each birthday for at least 20 years following her retirement (the first withdrawal will be on her 66th birthday). Your friend intends to invest her money in the local savings bank which offers 4 percent per year. She wants to make equal annual deposits on each birthday in a new savings account she will establish for her retirement fund.

If she starts making these deposits on her 36th birthday and continues to make deposits until she is 65 (the last deposit will be on her 65th birthday), what amount must she deposit annually to be able to make the desired withdrawals upon retirement?
Question 6 (30 points):

The current level of the S&P 500 is 1040. The risk-free interest rate per year is 2%. Assume negligible dividends. The 6 month futures contract is trading at 1060.

(a)(15 points) Is there an arbitrage opportunity? Briefly explain.
Question 7 Continued:

(b)(15 points) If there is an arbitrage opportunity, what strategy would you use to exploit it without using any funds of your own?