Discount rates

Class 12
Financial Management, 15.414
Today

Discount rates

- Using the CAPM
- Estimating beta and the cost of capital

Reading

- Brealey and Myers, Chapter 9
- Graham and Harvey (2000, p. 1 – 10)
Review

The CAPM

➤ **Measuring risk**
A stock’s systematic risk is measured by beta, the slope when the stock return is regressed on the market:

\[
R_i = \alpha + \beta R_M + \varepsilon
\]

➤ **Required returns**
Investors should be compensated for bearing non-diversifiable, beta risk. The required return on a stock is:

\[
E[R_i] = r_f + \beta_i E[R_M - r_f]
\]

**Market risk premium**
The risk-return trade-off

\[ \text{Slope} = E[R_M] - r_f \]

Stock's expected return vs. Stock's beta

- \( \beta = 0 \)
- \( \beta = 0.5 \)
- \( \beta = 1.5 \)

Market portfolio (\( \beta = 1 \))
Using the CAPM

Valuation

\[
NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \ldots
\]

Discount rate
The rate of return that investors demand on investments with the same level of risk.

CAPM

➢ Risk = the project’s beta

➢ Discount rate = \( r_f + \beta_{\text{project}} \ E[R_M - r_f] \)
Using the CAPM

Practical issues

1: How can we estimate the project’s beta?

2: What is the riskfree rate and the market risk premium?

3: How does debt affect risk and the cost of capital?

4: Additional risk factors?
Example

It’s 1979. Southwest Airlines, a growing start-up, has been profitable as the low-cost airline in the Texas market. Southwest is thinking about expanding to other U.S. cities. Management forecasts that the expansion will cost $100 million over the next few years but will lead to strong future growth ($ millions):

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>49.0</td>
<td>81.1</td>
<td>136.1</td>
<td>213.1</td>
<td>270.4</td>
<td>331.2</td>
</tr>
<tr>
<td>NI</td>
<td>7.5</td>
<td>17.0</td>
<td>16.7</td>
<td>28.4</td>
<td>34.2</td>
<td>34.0</td>
</tr>
<tr>
<td>NWC</td>
<td>5.1</td>
<td>9.7</td>
<td>10.7</td>
<td>12.4</td>
<td>11.1</td>
<td>19.3</td>
</tr>
<tr>
<td>CAPX</td>
<td>41.5</td>
<td>45.1</td>
<td>54.5</td>
<td>56.7</td>
<td>79.4</td>
<td>140.2</td>
</tr>
</tbody>
</table>

Growth is expected to slow to 10% annually after 1982.

What cost of capital should Southwest use to evaluate the proposed expansion?
Southwest stock price, 1970 – 1979
Issue 1

How can we estimate the project’s beta?

What factors are important?

➢ Two approaches
  Estimate the firm’s beta
  Estimate the industry’s beta (comparables)

➢ How much data?
  5 – 10 years of monthly data
Estimating beta

1: Estimate the firm’s beta

➢ Advantage

If the project has the same risks as the firm (an expansion), this approach measures exactly what we want

➢ Disadvantages

Generally not very precise (high standard error)

Firm’s beta might change over time

Can’t be used for projects in a new line of business or for diversified firms
Southwest

Is this approach useful for SW?

- Is the risk (beta) of the expansion likely to be the same as the beta of the firm?

- Is Southwest’s past beta likely to be a useful guide for the future beta of the project?

Southwest, 1973 – 1979 (84 months)

\[ R_{SW} = \alpha + \beta_{SW} R_M + \varepsilon_i \]

Estimate: \( \beta_{SW} = 1.25 \) (std error = 0.31); \( R^2 = 0.16 \)

\[ R_M = \text{return on a market index, like S&P 500} \]
Southwest vs. Total U.S. market return

Slope = 1.25
Southwest’s beta over time

1973-1977: $\beta = 1.20$

1978-1982: $\beta = 1.14$

1983-1987: $\beta = 1.06$

1988-1992: $\beta = 1.52$
Estimating beta

2: Estimate the industry’s beta*

➢ Advantages

Beta estimated more precisely.

Appropriate if the project is in a new line of business.

➢ Disadvantages

Do the firm’s really have the same risk as the project?

Do they serve different markets? Do they have more debt? Do they have the same cost structure?

* Estimate the betas of individual firms and then average, or estimate the beta of an industry portfolio.
Southwest

Is this approach useful for SW?

Is the risk (beta) of the expansion likely to be the same as the beta of other airlines?

Airline betas, 1973 – 1979

<table>
<thead>
<tr>
<th>Airline</th>
<th>β</th>
<th>Airline</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>1.42</td>
<td>Northwest</td>
<td>1.35</td>
</tr>
<tr>
<td>Continental</td>
<td>1.18</td>
<td>United</td>
<td>1.55</td>
</tr>
<tr>
<td>Delta</td>
<td>1.30</td>
<td>USAir</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Average = 1.36, standard error of 0.13
Airline industry vs. Total U.S. market return

Slope = 1.36
**Issue 2**

**Riskfree rate?**

\[ r_{\text{project}} = r_f + \beta_{\text{project}} (E[R_M] - r_f) \]

Should the riskfree rate be the short-term Tbill rate or the long-term Tbond rate?

**Match horizons**

If short-lived project, use Tbill rate
If long-lived project, use Tbond rate (say, 10-year)

**Riskfree rate changes a lot over time**

1979: Tbill rate = 9.65%, Tbond rate = 10.39%
2003: Tbill rate = 0.93%, Tbond rate = 4.31%
Interest rates, 1953 – 2001

1-yr Tbill
10-yr Tbond
Issue 2

Market risk premium?

\[ r_{\text{project}} = r_f + \beta_{\text{project}} E[R_M - r_f] \]

Historical estimates

- 1872 – 1999: 5.73% (std error = 1.63%)
- 1926 – 1999: 8.26% (std error = 2.24%)
- 1963 – 1999: 6.44% (std error = 2.51%)

\[ r = DY + g \]

- 1872 – 1999: 3.64% (std error = 1.15%)
- 1872 – 1949: 3.79% (std error = 1.78%)
- 1950 – 1999: 3.40% (std error = 0.99%)

Going forward? My guess, 4 – 6%
Market risk premium

Survey of CFOs

Source: Graham and Harvey, 2002
Southwest

Cost of capital

Firm’s beta: $\beta_{SW} = 1.25$

Industry’s beta: $\beta_{Airlines} = 1.36$

Riskfree rate = Tbond rate = 10.39%

Market risk premium = 5.0%

Discount rate*

$r = r_f + \beta_{project} \ E[R_M - r_f] = 10.39 + 1.30 \times 5.00 = 16.89%$

* If no debt
Issue 3

Debt financing, part 1

If the firm has debt, the cost of capital (discount rate) is a weighted average of the costs of debt and equity financing.

Cost of equity: \( r_E = r_f + \beta_E \cdot E[R_M - r_f] \)

Cost of debt:

1. \( r_D = r_f + \beta_D \cdot E[R_M - r_f] \)
2. \( r_D = \text{yield on the firm’s bonds} \)

After-tax weighted average cost of capital

\[
WACC = \frac{D}{A} (1 - \tau) r_D + \frac{E}{A} r_E
\]
Balance sheet

Assets

- Current Assets

Fixed Assets

1. Tangible fixed assets
2. Intangible fixed assets

Liabilities and Equity

- Current Liabilities
- Long-Term Debt
- Shareholders’ Equity
Southwest

In 1979, Southwest was financed with 20% debt (debt / firm value). The borrowing rate was 11.4% and the tax rate was 35%. What is Southwest’s WACC?

- **Cost of equity**
  \[ \beta_E = 1.30 \Rightarrow r_E = 10.39 + 1.30 \times 5.00 = 16.89\% \]

- **Weighted-average cost of capital**
  \[ WACC = 0.20 \times (1 - 0.35) \times 11.4\% + 0.80 \times 16.89\% = 14.9\% \]

- **Discount rate = 14.99\%**
Debt financing, part 2

If firms have different debt ratios, we cannot directly compare the stock betas of firms in the same industry.

**Firms with higher leverage should have riskier equity**
Higher D/V → higher $\beta_E$
Complicates the use of industry betas.

1. Estimate equity betas for each firm
2. Calculate $r_E$ and WACC for each firm
3. Use the industry’s WACC to estimate the cost of capital for the project
Southwest

Airline industry

Equity betas

<table>
<thead>
<tr>
<th>Airline</th>
<th>$\beta_E$</th>
<th>Airline</th>
<th>$\beta_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>1.42</td>
<td>Northwest</td>
<td>1.35</td>
</tr>
<tr>
<td>Continental</td>
<td>1.18</td>
<td>United</td>
<td>1.55</td>
</tr>
<tr>
<td>Delta</td>
<td>1.30</td>
<td>USAir</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Leverage ratios

<table>
<thead>
<tr>
<th>Airline</th>
<th>D/V</th>
<th>Airline</th>
<th>D/V</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>42%</td>
<td>Northwest</td>
<td>22%</td>
</tr>
<tr>
<td>Continental</td>
<td>30%</td>
<td>United</td>
<td>37%</td>
</tr>
<tr>
<td>Delta</td>
<td>53%</td>
<td>USAir</td>
<td>25%</td>
</tr>
</tbody>
</table>
Southwest

The tax rate is 35%, $r_D = 11.4\%$, $r_f = 10.39\%$, and $E[R_M - r_f] = 5.0\%$.

$$r_E = r_f + \beta_E E[R_M - r_f]$$

$$WACC = \frac{D}{A} (1 - \tau) r_D + \frac{E}{A} r_E$$

<table>
<thead>
<tr>
<th>Airline</th>
<th>$\beta_E$</th>
<th>$r_E$</th>
<th>D/A</th>
<th>WACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>1.42</td>
<td>17.5%</td>
<td>42%</td>
<td>13.3%</td>
</tr>
<tr>
<td>Continental</td>
<td>1.18</td>
<td>16.3%</td>
<td>30%</td>
<td>13.6%</td>
</tr>
<tr>
<td>Delta</td>
<td>1.30</td>
<td>16.9%</td>
<td>53%</td>
<td>11.9%</td>
</tr>
<tr>
<td>Northwest</td>
<td>1.35</td>
<td>17.1%</td>
<td>22%</td>
<td>15.0%</td>
</tr>
<tr>
<td>United</td>
<td>1.55</td>
<td>18.1%</td>
<td>37%</td>
<td>14.2%</td>
</tr>
<tr>
<td>USAir</td>
<td>1.37</td>
<td>17.2%</td>
<td>25%</td>
<td>14.8%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>1.36</strong></td>
<td><strong>17.2%</strong></td>
<td><strong>35%</strong></td>
<td><strong>13.8%</strong></td>
</tr>
</tbody>
</table>
Issue 4

Multifactor models

Beta might not fully summarize all relevant risks. Additional risk factors could be important.

**Measuring risk**
Regress $R_i$ on macroeconomic risk factors, $F_1 \ldots F_K$

$$R_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \ldots + \beta_{iK} F_K + \epsilon_i$$

$\beta_{ik}$ is firm i’s sensitivity to the factor.

**Expected returns**
Expected returns are linearly related to risk

$$E[R_i] = \gamma_0 + \gamma_1 \beta_{i1} + \gamma_2 \beta_{i2} + \ldots + \gamma_N \beta_{iN}$$

$\gamma_k$ is the risk premium for factor k.
Multifactor models

Fama-French 3-factor model*

- CAPM misses risk factors associated with size and B/M
- What are the risks?

\[ R_M = \text{Market portfolio return} \]
\[ \text{SMB} = \text{Small stock return} - \text{Big stock return} \]
\[ \text{HML} = \text{High-B/M stock return} - \text{Low-B/M stock return} \]

\[ R_i = \alpha_i + \beta_i R_M + s_i R_{SMB} + h_i R_{HML} + \varepsilon_i \]

\[ E[R_M - r_f] \approx 5.0\%, \quad E[R_{SMB}] \approx 3.0\%, \quad E[R_{HML}] \approx 4.0\% \]

*http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/
<table>
<thead>
<tr>
<th>Decile</th>
<th>$\beta$</th>
<th>$R^2$</th>
<th>Decile</th>
<th>$\beta$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low B/M</td>
<td>1.10</td>
<td>0.88</td>
<td>Smallest</td>
<td>1.33</td>
<td>0.56</td>
</tr>
<tr>
<td>2</td>
<td>1.08</td>
<td>0.90</td>
<td>2</td>
<td>1.06</td>
<td>0.73</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>0.92</td>
<td>3</td>
<td>1.13</td>
<td>0.79</td>
</tr>
<tr>
<td>4</td>
<td>0.99</td>
<td>0.89</td>
<td>4</td>
<td>1.14</td>
<td>0.84</td>
</tr>
<tr>
<td>5</td>
<td>0.91</td>
<td>0.87</td>
<td>5</td>
<td>1.14</td>
<td>0.86</td>
</tr>
<tr>
<td>6</td>
<td>0.86</td>
<td>0.84</td>
<td>6</td>
<td>1.10</td>
<td>0.88</td>
</tr>
<tr>
<td>7</td>
<td>0.93</td>
<td>0.76</td>
<td>7</td>
<td>1.04</td>
<td>0.91</td>
</tr>
<tr>
<td>8</td>
<td>1.04</td>
<td>0.74</td>
<td>8</td>
<td>1.10</td>
<td>0.93</td>
</tr>
<tr>
<td>9</td>
<td>1.16</td>
<td>0.64</td>
<td>9</td>
<td>1.00</td>
<td>0.96</td>
</tr>
<tr>
<td>High B/M</td>
<td>1.29</td>
<td>0.54</td>
<td>Largest</td>
<td>0.90</td>
<td>0.97</td>
</tr>
</tbody>
</table>
Southwest Airlines

Cost of capital

\[ R_{SW} = \alpha + \beta_{SW} R_M + s_{SW} R_{SMB} + h_{SW} R_{HML} + \varepsilon_i \]

\[ \hat{\beta}_{SW} = 1.123 \]
\[ \hat{s}_{SW} = 0.623 \]
\[ \hat{h}_{SW} = 0.442 \]

Cost of equity

\[ r_E = 10.4 + 1.123 \times 5.0 + 0.623 \times 3.0 + 0.442 \times 4.0 = 19.7\% \]

WACC

\[ WACC = 0.20 \times (1 - 0.35) \times 11.4\% + 0.80 \times 19.7\% = 17.2\% \]