Introduction

Credit derivatives are financial instruments whose payoffs are linked to certain credit events.

A commonly stipulated credit event is default by a named issuer, which could be a corporation or a sovereign issuer.

More generally, credit events can be defined in terms of downgrade, merger, cross default, failure to pay, restructuring, etc.

According to a survey by the British Bankers’ Association, the credit derivatives market accounted for an estimated $586 billion at the end of 1999. It is predicted to reach $1.58 trillion by 2002.

Some key credit derivative instruments: credit swap, total return swaps, collateralized debt obligations, spread options, etc.
Credit Swap

$100 - Y(\tilde{\tau})$

0 1 2 3 4 5 \tilde{\tau} \tau

U

Figure 1: Cash flows of a credit swap, buyer of a credit swap.

$T$: the random credit event time.

$\tau$: the maturity of the credit swap.

$U$: annuity of the credit swap.

$100$: par value of the underlying.

$Y(t)$: time t market value of the underlying.
Use, Credit Swap to Mitigate Credit Risk

Credit risk often emerges whenever a corporation enters into long term business relationships with large customers or suppliers.

For example, a credit exposure arises when a company has a long term product delivery contract with a customer in exchange for a future cash-flow stream.

This exposure can be particularly pronounced if a large upfront investment is required.

To mitigate such credit risk, the company can purchase a credit swap, whose payoff is linked to the credit event of its customer.
Credit Swap Valuation

Define the credit event time as the time when firm XYZ defaults. Use the model of constant default intensity:

For simplicity, let’s assume risk neutral pricing, and constant risk free rate $r$. Let $a_t(\lambda)$ be the time $O$ value of receiving $\$ 1$ at the end of year $t$ in the event that default is after that date:

Let $-b_t(\lambda)$ be the time $O$ value of receiving $\$ 1$ at the end of year $t$ in the event that default happens in year $t$:

\[
b_t(\lambda) = e^{-r} \cdot Prob(t - 1 \leq T < t) \]

\[
= e^{-r \cdot t} \left( e^{-\lambda(t-1)} - e^{-\lambda \cdot t} \right) \quad (2)
\]

The time $O$ value of an annuity of $\$ 1$ paid at the end of each year until default or maturity $\tau$, whichever comes first, is

\[
A_0 = a_1(\lambda) + a_2(\lambda) + \cdots + a_\tau(\lambda) \quad (3)
\]

The time $O$ value of a $\$ 1$ payment made at the end of the firm’s default year, provided the default date is before the maturity date $\tau$, is

\[
B_0 = b_1(\lambda) + b_2(\lambda) + \cdots + b_\tau(\lambda) \quad (4)
\]

Suppose that the loss of face value at default carries no risk premium and, has an expected value of $f$. The time $0$ value of the default swap is

\[
V_0 = B_0 \cdot f - A_0 \cdot U \quad (5)
\]

for any given annuity $U$. 

The default swap spread \( U \) is obtained by solving \( V_0 = 0 \), leaving:

\[
U = \frac{B_0 f}{A_0}
\]  

(6)
Collateralized Debt Obligations

A CDO is an asset backed security whose underlying collateral is typically a portfolio of bonds (corporate or sovereign) or bank loans.

A CDO cashflow structure allocates interest income and principal repayments from a collateral pool of different debt instruments to a prioritized collection (tranches) of CDO securities.

Figure 2: CDO-relationships.
Why CDO?

In perfect capital markets, CDOs would serve no purpose; the costs of constructing and marketing a CDO would inhibit its creation.

In practice, CDOs address some important market imperfections.

First, banks and certain other financial institutions have regulatory capital requirements that make it valuable for them to securitize and sell some portion of their assets, reducing the amount of (expensive) regulatory capital that they must hold.

Second, individual bonds or loans may be illiquid, leading to a reduction in their market values. Securitization may improve liquidity, and thereby raise the total valuation to the issuer of the CDO structure.
The Balance Sheet CDO

FleetBoston plays "Good Bank-Bad Bank," unloading $1.35 Billion in troubled loans.

Patriarch Partners LLC, a NY fund management boutique, created a CLO to raise about $1 billion to acquire the loans.

See the following figure how FleetBoston’s problem loans made their way from Fleet’s books to a special collateralized-debt obligation, funded by investors.

\[
C(x_t, \lambda_t) \approx \text{delta} - \text{gamma} \text{ in } (x_t, \lambda_t)
\]

\[
C(x_0, \lambda_0) \quad \text{survive} \quad p(\lambda_0)
\]

\[
1 - p(\lambda_0)
\]

\[
\text{default}
\]

\[
RC(x_0, \lambda_0) \quad \text{(recovery fraction R)}
\]

\[
C(x, \lambda) = \text{value of defaultable position at underlying } x, \text{ default intensity } \lambda
\]

Figure 3: Setting-up a special purpose vehicle.
Economic Principles of Credit Risk

Several Economic principles distinguish credit risk from other types of market risk, and might thereby influence the choice of risk measurements and techniques for risk mitigation:

- Adverse selection: setting credit exposure limits;
- Winner’s curse: setting limits on credit risk concentrations (by industry, by geographic location, by credit rating, etc.);
- Moral hazard: limit borrower’s access to credit.
Adverse Selection

There may be a significant amount of private information regarding the credit quality of a junk bond or a bank loan. An investor may be concerned about being "picked off" when trading such instruments.

Given the risk of being picked off, the buyer offers a price that, on average, is below the price at which the asset would be sold in a setting of symmetric information.

This reduction in price due to adverse selection is sometimes called a "lemon’s premium".
Mitigating Adverse Selection

In general, adverse selection cannot be eliminated by securitization of assets in a CDO, but it can mitigated.

The seller achieves a higher total valuation (for what is sold and what is retained) by designing the CDO structure so as to concentrate the "lemon’s" problem into small subordinate tranches, leaving the large senior tranche relatively immune to the effects of adverse selection.

The large senior tranche can therefore be sold at a small lemon’s premium.

The issuer can retain significant fractions of smaller subordinate tranches that are more subject to adverse selection.
Credit Switches

While both single name credit default swaps and synthetic securitizations are effective in transferring credit risk from one risk taker to another, there is often significant cost associated with these transactions.

Instead of eliminating the credit exposures, a corporation can retain the same amount of credit risk, but use zero cost credit switches to diversify the credit portfolio.

For example, one can use credit switches to diversify credit risks across different industries.
Measuring Credit Risk

For the purpose of measuring features of the distribution of P&L, credit risk should be viewed and computed as part of market risk.

At the same time, the measurement of credit risk provides its own set of challenges because many credit sensitive instruments are relatively illiquid, remain on a firm’s books, and cannot be reliably marked to market without modeling the default process.

Moreover, the problems of moral hazard and adverse selection recommend the use of limits on the bilateral exposures to counterparties.
Specialized Measures of Credit Risk

There are reasons (e.g., adverse selection and moral hazard) to track credit risk, by counterparty, that go beyond the contribution made by credit risk to overall market risk. To better understand the nature of the exposures to credit risk, risk managers have explored several complementary measures of credit risk, including:

- Market value of default loss
- Exposure: the risk that one’s exposure to a given counterparty (or within a given concentration area) excess a given threshold.

In addition to these, and other, internal measures of credit risk, financial institutions are required to measure and report credit risk to the appropriate regulatory authorities.
Measuring Credit Exposures

Regulators require financial institutions to measure both current and potential future exposure. Guidelines are set forth by both the DPG and BIS. For example, the DPG guidelines provide the following definitions:

- Current Exposure: $X_t = Y_t^+$ for a position $t$ with market value $Y_t$, where $Y_t$ is the combination of all positions subject to bi-lateral netting agreements with one counterparty.

- Potential Exposure: the 99% worst case exposure.

- Aggregating Credit Risks: adding current and potential exposures, and then multiplying by an applicable default ratio for the counterparty.
Exposure Disclosure

The DPG recommends reporting of:

- the top 20 credit exposures, by counterparty;
- aggregate credit risk organized by the 3 categories "net replacement value," gross replacement value," and "new exposure";
- aggregate credit risk organized by the 3 classes "credit rating of counterparty", "counterparty industry sector" and "counterparty geographic location".

Large broker dealers of OTC derivatives commonly report, to the public, the current replacement cost of their portfolios as a measure of credit risk.
Credit Risk Value and VaR

One can extend the methodology for estimation of portfolio market risk measures, such as VaR, so as to include credit risk:

- Catalog the sensitivities of all positions to fluctuations in market value due to 1. changes in underlying market wide risk factors (such as prices, rates, market credit spreads, and volatilities) 2. counterparty specific fluctuations in credit quality
- Applying a risk model for changes in market wide risk factors and counterparty specific credit quality, estimate the overall risk all positions, allowing for correlation.
Modeling Correlated Defaults

There are N counterparties, each with some default intensity $\lambda_i$.

We dial in correlations between default intensity by allowing for common credit events, arriving according to some intensity process $\lambda_c$.

At the arrival of a common credit event, all counterparties are exposed (with probability $p$) to immediate default.

Effectively, there are two components to the default intensity of counterparty $i$:

$$\lambda_i = \lambda_i^I + p\lambda^c$$

(7)

where $\lambda_i^I$ is the default intensity specific to counterparty $i$. 
Non Financial Firms

Non financial firms can manage and mitigate their exposures to the credit quality of their customer, suppliers, or clients through a variety of credit derivatives.

- Credit Swaps;
- Total Return Swaps;
- Credit Switches.
Focus:

- Reyfman and Toft (2001)
Preparation for Next Class

Please read:

- BKM Chapter 12,
- Rubinstein (2001)
- Daniel and Titman (1999) and