Class 6: Inventory Lecture

Design Decision:
CAPACITY (cf. Class 3)

Planning/Control Decision:
INVENTORY

(Oct 2001: $1.16 trillion in US!)

Trade-off: Inventory Cost Vs. Service Level
From the Trenches...

Too much:


Too little:


• “Since 1990 we have designated the Department of Defense’s management of its inventory, including spare parts, as high risk because [...] its management systems and procedures were ineffective.” – US General Accounting Office. “Army Inventory: Parts Shortages Are Impacting Operations and Maintenance Effectiveness,” August 2001.
Why Inventory Costs Money

- Cost of (stuck) capital
- Obsolescence
- Storage
- Insurance
- Security
- Theft (Shrinkage)

Typical per annum inventory holding cost:
Financial Inventory Metrics

\[
\text{Inventory Turns} = \frac{\text{Inventory Value} \times \text{Holding Cost}}{\text{COGS}}
\]

\[
\text{Inventory Cost / Unit} = \frac{\text{Inventory Value} \times \text{Holding Cost}}{\text{COGS}}
\]

Example: 10k filings, 2002 ($M)

<table>
<thead>
<tr>
<th></th>
<th>Wal Mart Stores Inc.</th>
<th>Kmart Corp.</th>
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<tbody>
<tr>
<td>Inventory</td>
<td>$22,749</td>
<td>$4,825</td>
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<td>C.O.G.S</td>
<td>$171,562</td>
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<td>Type of Inventory</td>
<td>Decision Tool</td>
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<td>Safety Inventory</td>
<td>Newsboy Model</td>
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<td>Cycle Inventory</td>
<td>EOQ Model</td>
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<td>Seasonal Inventory</td>
<td>Buildup Diagram</td>
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<td>Speculative Inventory</td>
<td>Finance</td>
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<td>In-Process/Pipeline Inventory</td>
<td>Little’s Law</td>
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<td>Marketing/Shelf Inventory (Retail)</td>
<td>Experience</td>
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Economic Order Quantity Model

• Set order size for repetitive ordering process with fixed order cost

• Trade-off:
  – Order size too large (too much average inventory) versus
  – Order size too small (too much ordering cost)

• Examples:
  – Ordering/Inventory replenishment policy;
  – Batch size on machine with setup time…
Running to the Store a Lot...

Mon □ Tue □ Wed □ Thu □ Fri □ Sat □ Sun □

Inventory

2002 - Jérémie Gallien

Slide courtesy of Prof. Thomas Roemer, MIT.
...Vs. Running to the Store a Little

Inventory

Monday □ Tuesday □ Wednesday □ Thursday □ Friday □ Saturday □ Sunday □
EOQ Model Parameters

- **Q** = Order Quantity \( \text{decision} \)
- **D** = Demand Rate (units/time)
- **C** = Purchasing Cost ($/unit) \( \text{parameters} \)
- **F** = Fixed Order Cost ($)
- **H** = Inventory Holding Cost (% p.a.)

**Assumptions:**
- constant, deterministic demand
- instantaneous replenishment
EOQ Model Derivation

- **Inventory Cost** \( H \cdot \frac{C \cdot Q}{2} \); **Order Cost** \( F \cdot \frac{D}{Q} \);

- **Total Cost** \( V = F \cdot \frac{D}{Q} + C \cdot H \cdot \frac{Q}{2} \)

Slide courtesy of Prof. Thomas Roemer, MIT.
EOQ Formula

• Set first derivative to 0:
  \[ \frac{\partial V}{\partial Q} = - \frac{DF}{Q^2} + \frac{CH}{2} = 0 \]

• This yields:
  \[ Q^* = \sqrt{\frac{2 \cdot DF}{CH}} \]
EOQ Example

A PC assembly operation procures its 128Mb memory chips at $45 each (purchase + shipment cost) from a foreign vendor; in addition each order also costs $500 in customs fees. Assuming a constant demand of 400 chips per week and an inventory holding cost of 45%, how often would you order?
Newsvendor Model

• One time decision under uncertainty

• Trade-off:
  – Ordering too much (waste, salvage value < cost) versus
  – Ordering too little (excess demand is lost)

• Examples:
  – Restaurant;
  – Fashion;
  – High Tech;
  – Inventory decisions…
Christmas Tree Problem

DECEMBER

1 2 3 4 5 6 7
8 9 10 11 12 13 14
15 16 17 18 19 20 21
22 23 24 25 26 27 28
29 30 31

$100

Slide courtesy of Prof. Thomas Roemer, MIT.
Ordering Too Many...

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Slide courtesy of Prof. Thomas Roemer, MIT.
...Versus Ordering Too Few
Newsvendor Model Parameters

- $q =$ Order Quantity \quad \textit{decision}
- $c =$ Unit Cost
- $r =$ Unit Revenue \quad \{ \text{parameters} \quad (r > c > b) \}
- $b =$ Unit Salvage Value
- $d =$ Demand (unknown) \quad \textit{random variable}
Newsboy Objective

IF \( d > q \)
(demand > quantity ordered)

Opportunity cost:
\((r - c) \times (d - q)\)

If \( q > d \)
(quantity ordered > demand)

Disposal cost:
\((c - b) \times (q - d)\)

Objective:
minimize expected opportunity + disposal cost
Model Derivation

- IF \( d > q \)  
  (demand > order qty)

  **Profit:** \( q \cdot (r - c) \)

  **Incremental Analysis:** \( q \rightarrow q + 1: \)

  \[ \Delta \text{Profit:} \quad r - c \]

  **EAP:** 
  \[ P(d > q) \cdot (r - c) + P(d \leq q) \cdot (b - c) \]

- IF \( d < q \)  
  (demand < order qty)

  **Profit:** \( d \cdot (r - c) + (q - d) \cdot (b - c) \)

As long as the *Expected Additional Profit* [EAP] is positive, it is lucrative to increase \( q \) to \( q + 1 !!!\)

Slide courtesy of Prof. Thomas Roemer, MIT.
Newsvendor Formula

\[
P(d < q^*) = \frac{r - c}{r - b} = \frac{r - c}{(r - c) + (c - b)} = \frac{u}{u + o}
\]

Remark: If \( d \) is Normal(\( \mu, \sigma \)),

\[q^* = \mu + k.\sigma\] with

- \( \alpha = 95\% \rightarrow k = 1.64 \)
- \( \alpha = 99\% \rightarrow k = 2.32 \)
- \( \alpha = 99.9\% \rightarrow k = 3.09 \)

Slide courtesy of Prof. Thomas Roemer, MIT.
Newsvendor Example

Based on forecasts and marketing studies you are expecting a total lifecycle demand $N(60,000;20,000)$ for a new product due to launch in the future. The product has a gross margin of $750$ and a liquidation/disposal cost (for unsold inventory) of $250$. Because of long lead-times you must commit orders to supplier for the entire product life-cycle now. How much should you order?
Continuous Review System

“order Q whenever inventory reaches R”

Inventory

Order Quantity Q

Order 1 placed

LT 1

Reorder Level R

EDDLT1

EDDLT

DDLT1

DDLT

Safety Stock

Order 1 received

LT 2

EDDLT2

DDLT2

Time

LT = Lead Time

EDDLT = Expected Demand During Lead Time

DDLT = (Actual) Demand During Lead Time

Slide courtesy of Prof. Thomas Roemer, MIT.
(R,Q) Parameters

“order Q whenever inventory reaches R”

- Set $Q$ as the EOQ solution
- Set $R$ as the newsboy solution:

$$P(\text{DDLT} < R) = \alpha$$

where $\alpha$ is a desired service level (e.g. 95%)

Example (cont’d): if weekly demand for 128Mb chips is in fact $N(400,80)$ and delivery time is 2 weeks, for a 95% service level:
Periodic Review System

“order back to S every T time units”

LT = Lead Time
T = Cycle Time or Review Period
DDLT = Actual Demand During Lead Time
Qi = Order Size
S = Order Up To Level

Order 1 placed
Order 1 received

Safety Stock

Slide courtesy of Prof. Thomas Roemer, MIT.
(S, T) Parameters

“order back to S every T time units”

- Set T as the EOQ solution divided by the demand rate
- Set S as the newsboy solution:

\[ P(DDLTRP < S) = \alpha \]

where:
- \( \alpha \) is the desired service level (e.g. 95%)
- DDLTRP = Demand During Lead-Time and Review Period
Safety Stock Formula

• Under periodic and review systems, safety stock $SS$ (under normally distributed demand) is given by:

$$SS = k \sigma$$

- fractile depending on service level, e.g.:
  - 95% $\rightarrow$ $k = 1.64$
  - 99% $\rightarrow$ $k = 2.32$
  - 99.9% $\rightarrow$ $k = 3.09$

- standard deviation of DDLT or DDLTRP

2002 - Jérémie Gallien
Class 6 Wrap-Up

1. Financial inventory metrics: inventory turns, per unit inventory cost

2. Functions of inventory: seasonal, cyclical, safety, speculative, pipeline, shelf

3. EOQ & newsboy models

4. Continuous and discrete replenishment policies, safety stock formula