3D Boundary Layers

1) Infinite Swept Wing

2) Swept and Tapered

3) Skin Theory (Quasi-3D)

4) Infinite Swept Wing

\[ U_{w0} \perp = U_{w0} \cos \Lambda \]
\[ U_{w0} \parallel = U_{w0} \sin \Lambda \]
\[ We(x, z) = U_{w0} \parallel \]

**TSL Equations**

\[ \frac{\partial \rho u_0}{\partial x} + \frac{\partial \rho v_0}{\partial y} + \frac{\partial \rho w_0}{\partial z} = 0 \]
\[ \frac{\partial u_0}{\partial z} = 0 \text{ by geometry} \]
\[ \rho u_0 \frac{\partial u_0}{\partial x} + \rho v_0 \frac{\partial u_0}{\partial y} + \rho w_0 \frac{\partial u_0}{\partial z} = -\rho u_0 \frac{\partial p}{\partial x} + \rho u_0 \frac{\partial \mu_0}{\partial x} + \rho u_0 \frac{\partial \mu_0}{\partial z} + \frac{\partial^2 u_0}{\partial y^2} \]
\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \rho \frac{\partial u}{\partial t} + \rho e \frac{\partial e}{\partial t} + \frac{\partial p}{\partial y} + \frac{\partial E}{\partial z} + \frac{\partial \epsilon}{\partial y}
\]

\[\Rightarrow \text{x - z momentum uncoupled}\]

Solve using F-S along x with count crossflow in z

\[\text{Strong 3D effects}\]

Friction BL from:

- sep. \(-\) BL gets squeezed (very neg \(\frac{\partial w}{\partial z}\))

- No coupling in 2D

- Turbulent coupling via Reynolds stress (weak)

\[B) \text{Coral Cross flow Approx}\]

\[\text{Curtissian Transformation from } (x, y, z) \rightarrow (x', y', z')\]

\[x' = \int \frac{u e}{\sqrt{g' e}} \, dx + \int \frac{u e}{\sqrt{g' e}} \, dz\]

\[z' = \int \frac{u e}{\sqrt{g' e}} \, dz - \int \frac{u e}{\sqrt{g' e}} \, dx\]

\[x' \text{ aligned with } \frac{\partial e}{\partial z}\]

Assume \(e = 0\), \(\frac{\partial e}{\partial z} = 0\)

\[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0\]

\[p \left[ \frac{u^2}{\partial x} + \frac{v^2}{\partial y} - u w \frac{\partial w}{\partial z} + u^2 \frac{\partial w}{\partial y} \right] = \rho \frac{\partial u}{\partial t} + \frac{\partial E}{\partial z} + \frac{\partial \epsilon}{\partial y}\]
\[ p \left[ \frac{u^2}{2} + v \frac{dv}{dy} + u \frac{d}{dy} \left( \frac{u}{2} z_1 - u w_1 z_1 \right) \right] = \rho E k_1 + \frac{\partial \theta}{\partial y} \]

- \( k_1, k_2 \) are coordinates and curvatures

\[ \Rightarrow \text{we have 2D problem along each } z' = \text{constant} \]

 Extra unknown: 
- momentum \( W \) (not small)
- \( z' \)-mom

2-D BC ships

**Approximation 2-D**

Flow along streamlines (parametric)

\[ m' = \int \frac{dz}{r} = \int \frac{\sqrt{dx'^2 + dr'^2}}{r} \]

meridional coordinate
The second cascade problem along streamwise

3D DBL eqn:

\( r(m'), \ b(m') \) - streamwise thickness

\[ \frac{\partial f}{\partial m'} \]

3D sweep & Taper

Local streamwise coordinates \((s, \eta), (\tilde{u}, \tilde{w})\)

\[ u = \tilde{u} \cos \alpha - \tilde{w} \sin \alpha \]
\[ w = \tilde{u} \sin \alpha + \tilde{w} \cos \alpha \]

\[ \cos \alpha = \frac{ue}{\tilde{u}e} \]
\[ \sin \alpha = \frac{ue}{\tilde{u}e} \]

\[ \tilde{u}e = 0.8 \sqrt{ue^2 + \tilde{w}e^2} \]
\[ \tilde{w}e = 0 \]

Transformed integral thickness
\[ \rho \varepsilon \delta^* \cdot \int (\varepsilon u^e - p^e) \, dy = \rho u \delta^* - \rho u \delta^* \]

Substitute transformed velocity

Similarly, the thickness \( \cdot \) (see transient)

Note profiles defined in local streamline coordinates / direction

The integral thickness in \((x, y)\) system can be calculated

Convenient integration coordinates are \( \xi, \eta \)

\[ \begin{align*}
\xi &= x \cos \lambda + z \sin \lambda \\
\eta &= -x \sin \lambda + z \cos \lambda
\end{align*} \]

Transform derivatives

\[ \frac{\partial}{\partial x} \left( \right) \]

\[ \frac{\partial}{\partial z} \]

Approximation in the case of infinite, yawed, tapered