Close Pass of a Target Planet

The inbound plane of relative motion is the plane containing the orbital velocity vectors of the spacecraft and the planet. The unit normal to that plane is

\[
i_n = \text{Unit}(\mathbf{v}_{SC} \times \mathbf{v}_P)
\]

so that the vector point of aim is

\[
\mathbf{r}_a = r_a i_n \times i_{\infty_i}
\]

The vector \( \mathbf{r}_a \) can be rotated about the direction of \( i_{\infty_i} \) so that the orientation of the passing plane, relative to the planet, can be controlled at will. The method for this is discussed in the following page.
The Rotation Matrix and the Rotation of a Vector

2.2

Let \( O \) be a fixed point on an axis whose direction is \( \mathbf{i}_\omega = l \mathbf{i}_x + m \mathbf{i}_y + n \mathbf{i}_z \) and let \( \mathbf{r} \) be the vector from \( O \) to a point \( P \). When the vector \( \mathbf{r} \) rotates about this axis, the point \( P \) moves in a circle of radius \( r \sin \alpha \) where \( \alpha \) is the angle between \( \mathbf{r} \) and \( \mathbf{i}_\omega \).

A small rotation through an angle \( \delta \psi \) produces a change \( \delta \mathbf{r} \) given by

\[
\delta \mathbf{r} = \delta \psi \mathbf{i}_\omega \times \mathbf{r} = \delta \psi \mathbf{S} \mathbf{r}
\]

where the constant matrix \( \mathbf{S} \) is skew-symmetric, i.e., \( \mathbf{S} = -\mathbf{S}^T \). Hence

\[
\frac{d\mathbf{r}}{d\psi} = \mathbf{S} \mathbf{r}
\]

is a linear vector differential equation for \( \mathbf{r} \) with constant coefficients whose solution is

\[
\mathbf{r} = e^{\psi \mathbf{S}} \mathbf{r}_0 \equiv (\mathbf{I} + \psi \mathbf{S} + \frac{1}{2!}\psi^2\mathbf{S}^2 + \frac{1}{3!}\psi^3\mathbf{S}^3 + \cdots) \mathbf{r}_0
\]

Also, since

\[
\mathbf{S} = \begin{bmatrix}
0 & -n & m \\
n & 0 & -l \\
-m & l & 0
\end{bmatrix}
\]

\[
\mathbf{S}^2 = \begin{bmatrix}
-m^2 - n^2 & lm & ln \\
lm & -l^2 - n^2 & mn \\
ln & mn & -l^2 - m^2
\end{bmatrix}
\]

\[
\mathbf{S}^3 = \begin{bmatrix}
0 & n & -m \\
-n & 0 & l \\
m & -l & 0
\end{bmatrix}
\]

we conclude that

\[
\mathbf{S}^3 = -\mathbf{S}
\]

Thus, all powers of the matrix \( \mathbf{S} \) are either \( \pm \mathbf{S} \) or \( \pm \mathbf{S}^2 \) and we have

\[
\mathbf{R} = \mathbf{I} + \left(\psi - \frac{1}{3!}\psi^3 + \cdots\right) \mathbf{S} + \left(\frac{1}{2!}\psi^2 - \frac{1}{4!}\psi^4 + \cdots\right) \mathbf{S}^2
\]

or

\[
\mathbf{R} = \mathbf{I} + \sin \psi \mathbf{S} + (1 - \cos \psi) \mathbf{S}^2
\]

Hence \( \mathbf{r} = \mathbf{R} \mathbf{r}_0 \) is equivalent to

\[
\mathbf{r} = \mathbf{r}_0 + \sin \psi \left(\mathbf{i}_\omega \times \mathbf{r}_0\right) + (1 - \cos \psi) \mathbf{i}_\omega \times (\mathbf{i}_\omega \times \mathbf{r}_0)
\]

Try this with \( \mathbf{r}_0 = \mathbf{i}_x \) and \( \mathbf{i}_\omega = \mathbf{i}_z \)

Application to Rotating the Point of Aim vector \( \mathbf{r}_a \) about the \( \mathbf{i}_{\infty i} \) direction

\[
\mathbf{r}_a = \frac{\mu}{2v^2 \sin^2 \nu} \mathbf{i}_{\infty i} \times (\mathbf{i}_{\infty i} \times \mathbf{i}_{\infty o}) \quad \text{See Problem 9–4}
\]

New \( \mathbf{r}_a = \mathbf{r}_a + \sin \psi (\mathbf{i}_{\infty i} \times \mathbf{r}_a) + (1 - \cos \psi) \mathbf{i}_{\infty i} \times (\mathbf{i}_{\infty i} \times \mathbf{r}_a) \)

16.346 Astrodynamics Lecture 12
Planetary FlyBys are definitely three-dimensional

The normal to the plane in which the spacecraft moves in its hyperbolic swingby of the planet is determined by the cross-product of the inbound and outbound relative velocity vectors.

\[ \mathbf{v}_{\infty o} \times \mathbf{v}_{\infty i} \]
The velocity components, which are part of this data, have never been published before. We give them below in units of feet per second:

Launch date: June 9, 1972

Outbound Earth velocity \([-6326 - 12628 - 5051]\) Magnitude = 15000 Ft/sec

Inbound Venus velocity \([28253 2698 335]\) Magnitude = 28384 Ft/sec

Outbound Venus velocity \([23564 15771 - 1274]\) Magnitude = 28383 Ft/sec

Inbound Mars velocity \([19624 - 17633 - 1790]\) Magnitude = 26443 Ft/sec

Outbound Mars velocity \([-20418 - 16713 1736]\) Magnitude = 26443 Ft/sec

Inbound Earth velocity \([-40370 - 7303 1836]\) Magnitude = 41066 Ft/sec

Return date: September 13, 1973

Launch date: February 6, 1966

Outbound Earth velocity \([-7907 - 13922 3985]\) Magnitude = 16499 Ft/sec

Inbound Venus velocity \([29837 1164 - 6313]\) Magnitude = 30520 Ft/sec

Outbound Venus velocity \([23529 18335 - 6453]\) Magnitude = 30519 Ft/sec

Inbound Mars velocity \([6061 - 14285 2957]\) Magnitude = 15797 Ft/sec

Outbound Mars velocity \([8659 - 13164 1115]\) Magnitude = 15796 Ft/sec

Inbound Earth velocity \([39120 512 2839]\) Magnitude = 39226 Ft/sec

Return date: December 17, 1967

Error correction in red was discovered by Steve Block (a student in 16.346) in the fall term of 2003 — over 42 years after the original posting!

The original velocity vector was incorrectly stated as \([16713 - 20418 1736]\).