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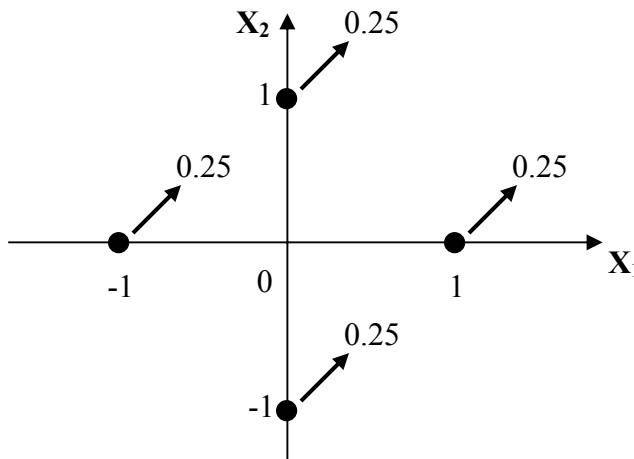
1.010 Uncertainty in Engineering  
Fall 2008

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**1.010 – Mini-Quiz #4**  
(40 min – open books and notes)

**Problem 1 (30 Points)**

Consider two variables  $X_1$  and  $X_2$ , each with possible values  $-1$  and  $1$ . The joint probability mass function of  $X_1$  and  $X_2$  is shown in the figure below. (Notice that the distribution is concentrated at four points, with equal probability  $0.25$  at each point)



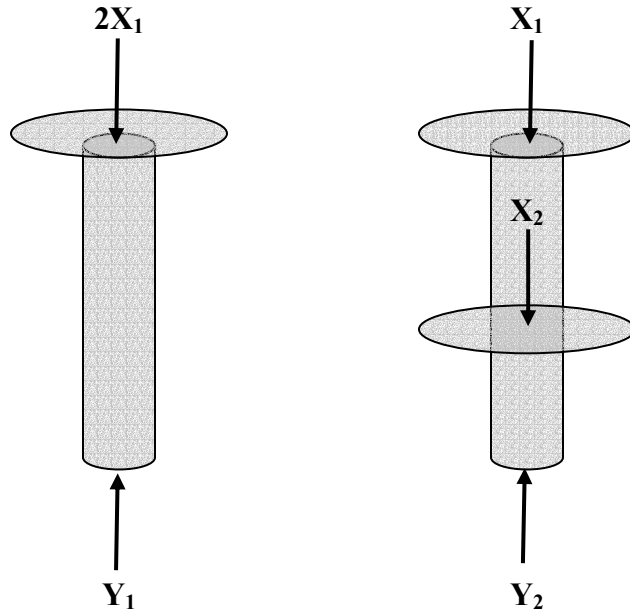
- (a) Are  $X_1$  and  $X_2$  independent?
  - (b) Are  $X_1$  and  $X_2$  uncorrelated?
- Justify your answers

**Problem 2 (30 Points)**

The time  $T$  ( in hours) for a car to travel 50 miles of a highway is given by  $T = \frac{50}{V}$ , where  $V$  is the car speed in miles/hour. Suppose that  $V$  has mean value  $m_V = 60$  miles/hour and standard deviation  $\sigma_V = 10$  miles/hour . Find in approximation the mean value and standard deviation of  $T$ .

### Problem 3 (40 Points)

Consider the two columns shown in the figure below, where  $X_1$  and  $X_2$  are random load variables.



The loads  $Y_1$  and  $Y_2$  at the base of the columns are given by:

$$Y_1 = 2X_1$$

$$Y_2 = X_1 + X_2$$

Suppose that  $X_1$  and  $X_2$  have common mean value  $m$  and common standard deviation  $\sigma$  and denote by  $\rho$  their correlation coefficient.

- Find the variances of  $Y_1$  and  $Y_2$ .
- What effect does  $\rho$  have on the variance of  $Y_2$ ?
- Explain in intuitive ways the results for  $\rho = -1$  and  $\rho = 1$ . Also compare with the variance of  $Y_1$ . (Notice that for  $X_1 = X_2$  the load on the two columns is the same).