1.022 Introduction to Network Models

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Lecture 1
Four other instructors contributing in putting together this course:

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Prerequisites

- **Courses:** 1.00 or 1.000; 18.03; 1.010 (or permission of instructor)

- **Mathematical maturity**
  - Proofs and deductive reasoning
  - Basic probability and statistics
  - Basic calculus and linear algebra

- **Programming**
  - We will use python for homework and final project
  - Recitation will go over basics
  - No need to know python in advance but ...
  - Need to know basic programming logic (conditional, loops, ...)

- **No knowledge on networks/graphs is required**
Homework, midterm, and final project

- **Homework** ⇒ 8 sets ⇒ 9 days for each
- 7 best out of 8 submissions will be considered
- Late homework will not be accepted
- We encourage collaboration but final version must be yours

- **Midterm** ⇒ in class
- Based on the first 12 lectures

- **Final Project** ⇒ Teams of 2-3 students
- How to control epidemics on graphs?
- Applied project plus competition module
- More information during week 4 ⇒ start early!

- Grading: Homework 30%, Midterm 30%, Final project 40%
Come to the lectures and take notes

⇒ Slides act as guide for blackboard discussions

Networks, Crowds, and Markets: Reasoning about a Highly Connected World
by David Easley and Jon Kleinberg

Can be downloaded at
Additional bibliography

- Material complemented with chapters from two additional books
  - Social and Economic Networks by Matthew O. Jackson
  - Networks: An Introduction by Mark Newman

- Occasionally, we will reference published journal papers
Pattern of interconnections among a set of “things”

Very general definition $\Rightarrow$ enormous range of topics

Growing public fascination with connectedness of modern society

Started with Euler’s 1735 solution of the Königsberg bridge problem.

Can you cross each bridge exactly once in a walk?

Complex system modeled as a mathematical network (graph)

Understanding systems at this level of abstraction

$\Rightarrow$ Powerful tool across disciplines

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The source map is public domain, modified by Bogdan Giuşcă.
Increasing interest

- **Network-based analysis has a long-standing history**
  - Study of Königsberg bridges (Euler, 1736)
  - Laws of electrical circuitry (Kirchoff, 1845)
  - Molecular structure in chemistry (Cayley, 1874)
  - Power grids (1910), telecommunications and the Internet (1960)

- **Explosion of interest in the last two decades**
  - Systems-level perspective in science
  - High-throughput data collection and computational power
  - Globalization and connected modern society
Network Science

- Study of complex systems through their network representations
  ⇒ Brain, society, economy, ...

- Universal terminology for describing complex systems
  ⇒ Consistent across areas of knowledge

- Ubiquity of networks redounds in high impact of Network Science
Early 20th century Hungarian poet and writer Frigyes Karinthy first came up with the idea that we live in a *small world*.

⇒ In a play, he suggested all 1.5Billion people of the earth are at most 5 acquaintances away from each other.

The sociologist Stanley Milgram made this famous in his study *The Small World Problem* (1967) — though this study is now largely discredited.

⇒ performed an experiment among residents of Wichita and Omaha to send a folder to a target, via acquaintances.

⇒ Consistent across areas of knowledge.

⇒ 42 of the 160 letters supposedly made it to their target, with a median number of intermediates equal to 5.5. Hence 6 degrees!

⇒ Yet his study was flawed ... why?
Huge potential impact

- Prediction of epidemics, e.g. the 2009 H1N1 pandemic

- Human Connectome Project to map the brain circuitry
Examples of networks

- **Increasing availability** of large, detailed network datasets

- **Different reasons** for wanting to study a particular network dataset
  - Care about the actual domain it comes from
  - Dataset as a proxy for a related network that is impossible to measure
  - Look for network properties that are common across different domains

- **One particular way of dividing the network datasets**
  - Collaboration graphs
  - Who-talks-to-whom graph
  - Information linkage graph
  - Technological networks
  - Networks in the natural world
**Collaboration graph**

- **Nodes** are mathematicians, **edges** denote co-authorship
  - **Paul Erdős** (~ 1500 papers) → central collaborative figure
  - **Erdős number** → distance from Erdős in this graph
  - Most scientists have Erdős number \( \leq 6 \) → **Small world phenomenon**

Graham, Ron. "A Portion of the Erdős Collaboration Graph." In Topics of Graph Theory. Edited by Frank Harary. New York Academy of Sciences, 1979. © New York Academy of Sciences. All rights reserved. This content is excluded from our Creative Commons license. For more information, see [https://ocw.mit.edu/help/faq-fair-use/](https://ocw.mit.edu/help/faq-fair-use/).
Who-talks-to-whom graph

- **Nodes** are employees of HP, **edges** denote email exchanges
  - Email interactions highly determined by hierarchy in company
  - Can be used to determine inter-departamental collaborations
  - What other insights can we get?


Nodes are political blogs, edges denote hyperlinks

⇒ Colors represent main political inclination of blog
⇒ Two clear clusters arise ⇒ Community detection
⇒ Can we recover these communities only from the network structure?

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Figure: A depiction of the map of Internet.
Figure: The web link structure centered at http://web.mit.edu (touchgraph)
Figure: The spread of an epidemic disease (such as the tuberculosis outbreak shown here) is another form of cascading behavior in a network. The similarities and contrasts between biological and social contagion lead to interesting research questions. (Andre et al. 2007)
Financial Network

Figure: The network of loans among financial institutions can be used to analyze the roles that different participants play in the financial system, and how the interactions among these roles affect the health of individual participants and the system as a whole. (Bech and Atalay 2008)
Topics that we will cover

- **Graph theory** ⇒ Study the structure of networks
  ⇒ Network models ⇒ How is network formation modeled?

- **Social network analysis**
  ⇒ Centrality measures ⇒ Who are the most important agents?
  ⇒ Community detection ⇒ Which are the main groups of agents?
  ⇒ Spectral graph theory and graph matrices

- **Dynamics and learning on networks**
  ⇒ Information spread and random walk on graphs
  ⇒ Propagation of epidemics on networks
  ⇒ Opinion formation from social interactions

- **Game theory** ⇒ Study the behavior on networks
  ⇒ Group of people must simultaneously choose how to act
  ⇒ Outcome depends on the actions of all the people
  ⇒ Payoffs, strategies, and equilibria ⇒ Counter-intuitive results