Lecture 5 - summary

Introduction of the continuum model

Three scales:

*Structural scale (H,B,D..) >> REV >> molecular scale*

The three scales are separated (">>" operator)

Goal: Derivation of equilibrium equations for REV $d\Omega$

Equilibrium:

$$\frac{d\phi}{dt} = \frac{d}{dt} \left( \rho \bar{V} d\Omega \right) \overset{def}{=} \bar{F}_{ext}$$

External forces:

$$\bar{F}_{ext} = \rho \bar{g} d\Omega + \sum_i \bar{T}_i d\omega_i$$

Continuum representative volume element REV

Definition of stress tensor (description of material forces only as function of position, not normal):

$$\sigma = \sigma_{ij} \epsilon_i \otimes \epsilon_j$$

$$\bar{T} \overset{def}{=} \bar{T}(\bar{x}^\alpha, \bar{n})$$

Complete problem (Dynamic Resultant + Moment Theorems):

**on $S$**:

$$\bar{T}(\bar{n}) + \bar{T}(-\bar{n}) = 0$$

**on $\partial \Omega$**:

$$\bar{T}^d = \bar{T}(\bar{n})$$

**in $\Omega$**:

$$\begin{cases}
\nabla \sigma + \rho (\bar{g} - \bar{a}) = 0 \\
\sigma_{ij} = \sigma_{ji} = 0 \text{ (static)}
\end{cases}$$

Local equilibrium:

$$\nabla \sigma + \rho (\bar{g} - \bar{a}) = 0$$

Skyscraper photograph courtesy of jochemberends on Flickr.