Using 
\[ \frac{\partial T}{\partial s^*} = \frac{\partial T}{\partial M} \frac{dM}{ds^*}, \]

We can re-write (18) as
\[ \frac{\partial T_o}{\partial M} \equiv - \frac{Ri_c}{r_t^2} \left( \frac{dM}{ds^*} \right). \] (19)

We can also re-write (6) as
\[ M_b = r_b^2 \left( \frac{1}{2} f - (T_b - T_o) \frac{dM}{ds^*} \right) \] (20)

Boundary layer entropy
\[ h \frac{ds_b}{dt} = C_k \left| V \right| \left( s_0^* - s_b \right) + C_d \frac{|V|^3}{T_b} \] (21)

Boundary layer angular momentum
\[ h \frac{dM}{dt} = -r \left| V \right| V \] (22)
Combine (21) and (22):

\[
\frac{ds_b}{dM} = - \frac{C_k}{C_D} \left( s_0^* - s_b \right) - \frac{|V|^2}{rV} - \frac{V_0^*}{T_b rV}
\]

Let \( s_b \approx s^* \), \( |V| \approx V \approx V_b \), \( r \approx r_b \)

\[
\rightarrow \frac{ds^*}{dM} = - \frac{C_k}{C_D} \left( s_0^* - s^* \right) - \frac{V_b}{r_b V_b} - \frac{V_b}{T_b r_b}
\]

Balance condition (8):

\[
\frac{V_b}{r_b} = - \left( T_b - T_o \right) \frac{ds^*}{dM}
\]
Eliminate $V_b$ between (23) and (24):

$$\left( \frac{ds^*}{dM} \right)^2 = \frac{T_b}{T_o} \frac{C_k}{C_D} r_b^2 \frac{\left( s_0^* - s^* \right)}{T_b - T_o}$$ (25)

Eliminate $r_b^2$ between (20) and (25):

$$\left( \frac{ds^*}{dM} \right)^2 + 2 \chi \frac{ds^*}{dM} - \frac{\chi f}{T_b - T_o} = 0,$$ (26)

where

$$\chi \equiv \frac{T_b}{T_o} \frac{C_k}{C_D} \frac{s_0^* - s^*}{2M}$$

Remember that

$$\frac{\partial T_o}{\partial M} \approx -\frac{\text{Ri}_c}{r_t^2} \left( \frac{dM}{ds^*} \right)$$ (19)
Integrate (26) and (19) inward from some outer radius $r_0$, defined such that

$$V = 0 \quad \text{at} \quad r = r_0$$

In general, integrating this system will not yield $T_o = T_t$ at $r = r_{max}$. Iterate value of $r_t$ until this condition is met.

If $V >> fr$, we ignore dissipative heating, and we neglect pressure dependence of $s_0^*$, then we can derive an approximate closed-form solution.
Assuming that $Ri$ is critical in the outflow leads to an equation for $T_o$ that, coupled to the interior balance equation and the slab boundary layer lead (surprisingly!) to a closed form analytic solution for the gradient wind (as represented by angular momentum, $M$, at the top of the boundary layer:

\[
\left( \frac{M}{M_m} \right)^2 \left( \frac{C_k}{C_D} \right) = 2 \left( \frac{r}{r_m} \right)^2 - \frac{C_k}{C_D} + \frac{C_k}{C_D} \left( \frac{r}{r_m} \right)^2 , \tag{27}
\]
The maximum wind speed, $V_m$, found from maximizing the radial dependence of wind speed in the solution (27) is given by

$$V_m^{2 - \frac{C_k}{C_D}} = V_p^2 \left( \frac{2r_m}{fr_o^2} \right)^{\frac{C_k}{C_D}}$$  \hspace{0.5cm} (30)
\[ V_p^2 \equiv \frac{C_k}{C_D} (T_b - T_t) (s_0^* - s_e^*) \]

Substituting (29) into (30) gives

\[ V_m^2 \approx V_p^2 \left( \frac{1}{2} \frac{C_k}{C_D} \right)^{\frac{C_k}{C_D}} \frac{1}{2 - \frac{C_k}{C_D}} \]  

(31)
Substituting (31) into (29) gives

\[ r_m \approx \left( \frac{1}{2} \right)^{\frac{3}{2}} \frac{fr_0^2}{\sqrt{(T_b - T_t)}(s_0^* - s_e^*)} \]

Also,

\[ r_t^2 = r_m^2 \frac{C_D}{C_k} Ri_c \]
Control

With dissipative heating
Numerical integrations with RE87 model (no dissipative heating, no pressure dependence of $k_0^*$) : Left, regular variables; Right: Velocity scaled by (31) and time scaled by the inverse square-root of the enthalpy exchange coefficient.
Effects of Pressure-Dependence of Surface Saturation Enthalpy
Maximum Wind Speed (m/s)

Hypercanes

$\mathcal{N} = 0.75 \quad C_k/C_D = 1.2$
Relationship between potential intensity (PI) and intensity of real tropical cyclones
Evolution with respect to time of maximum intensity
Evolution with respect to time of maximum intensity, normalized by peak wind
Evolution curve of Atlantic storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall.
Evolution curve of WPAC storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall.
CDF of normalized lifetime maximum wind speeds of North Atlantic tropical cyclones of tropical storm strength (18 m s\(^{-1}\)) or greater, for those storms whose lifetime maximum intensity was limited by landfall.
CDF of normalized lifetime maximum wind speeds of Northwest Pacific tropical cyclones of tropical storm strength (18 m s\(^{-1}\)) or greater, for those storms whose lifetime maximum intensity was limited by landfall.
Evolution of Atlantic storms whose lifetime maximum intensity was limited by landfall
Evolution of Pacific storms whose lifetime maximum intensity was limited by landfall
Composite evolution of landfalling storms

![Graph showing the time evolution of wind speeds for Atlantic and Pacific storms.](image-url)