14.123 Microeconomics III—Problem Set 1

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Instructions. You are encouraged to work in groups, but everybody must write their own solution to the problem that is for grade. Good Luck!

(i) (For Grade) There are $n$ individuals. Each individual $i$ has constant absolute risk aversion $\alpha_i > 0$ and an asset that pays $X_i$ where $(X_1, \ldots, X_n) \sim N((\mu_1, \ldots, \mu_n), \Sigma)$.

(a) What are the optimal risk sharing contracts? What is the vector of payoffs from an optimal risk-sharing contract? Characterize the set of the vectors of certainty equivalents from optimal risk sharing contracts.

(b) Answer (a) for $\alpha_1 = \cdots = \alpha_n$, $\mu_1 = \cdots = \mu_n$ and $\Sigma = \sigma^2 \begin{pmatrix} 1 & \rho & \cdots & \rho & \rho \\ \rho & 1 & \cdots & \rho & \rho \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho & \rho & \cdots & 1 & \rho \\ \rho & \rho & \cdots & \rho & 1 \end{pmatrix}$.

How much the society as a whole are willing to pay all of these assets? Assuming that they write a symmetric contract, what is the preference relation of an individual on $(\sigma^2, \rho)$ pairs? Briefly discuss.

(ii) Exercise 2.1 in lecture notes.

(iii) Consider the set of lotteries $(p_x, p_y, p_z)$ on the set of outcomes $\{x, y, z\}$ where $p_x, p_y,$ and $p_z$ are the probabilities of $x, y,$ and $z,$ respectively.

(a) For each (partial) preference below, determine whether it is consistent with expected utility maximization. (If yes, find a utility function; if no, show that it cannot come from an expected utility maximizer.)

i. $(0, 1, 0) > (1/8, 6/8, 1/8)$ and $(7/8, 0, 1/8) > (6/8, 1/8, 1/8)$

ii. $(1/4, 1/4, 1/2) > (3/4, 0, 1/4) > (5/6, 1/6, 0) > (1/2, 1/3, 1/6)$

(b) For each family of indifference curves below, determine whether it is consistent with expected utility maximization. (If yes, find a utility function; if no, show that it cannot come from an expected utility maximizer.)

i. $p_y = c - 2p_x$ (where $c$ varies)

ii. $p_y = c(p_x + 1)$ (where $c$ varies)
iii. \( p_y = c - 2\sqrt{px} \) (where \( c \) varies)

(c) Find a complete and transitive preference relation on the above lotteries that satisfies the independence axiom but cannot have an expected utility representation.

(iv) Alice has \( M \) dollars and has a constant absolute risk aversion \( \alpha \) (i.e. \( u(x) = -e^{-\alpha x} \)) for some \( \alpha > 0 \). With some probability \( \pi \in (0,1) \) she may get sick, in which case she would need to spend \( L \) dollars on her health. There is a health-insurance policy that fully covers her health care expenses in case of sickness and costs \( P \) to her. (If she buys the policy, she needs to pay \( P \) regardless of whether she gets sick.)

(a) Find the set of prices \( P \) that she is willing to pay for the policy. How does the maximum price \( P \) she is willing to pay varies with the parameters \( M, L, \alpha, \) and \( \pi \)?

(b) Suppose now that there is a test \( t \in \{-1, +1\} \) that she can take before she makes her decision on buying the insurance policy. If she takes the test and the test \( t \) is positive, her posterior probability of getting sick jumps to \( \pi^+ > \pi \) and if the test is negative, then her posterior probability of getting sick becomes 0. What is the maximum price \( c \) she is willing to pay in order to take the test? (Take \( P \leq \bar{P} \).)