Instructions. You are encouraged to work in groups, but everybody must write their own solutions. Each question is 33 points. Good Luck!

1. Problem 3 in Problem Set 2.

2. Bob has just retired and has \( w_0 \) dollars. His utility from a consumption stream \((c_0, c_1, \ldots)\) is
\[
\sum_{t=0}^{\infty} \delta^t u(c_t),
\]
where \( u : \mathbb{R} \to \mathbb{R} \) is a von Neumann-Morgenstern utility function with constant relative risk aversion \( \rho > 1 \). For each \( t \), he dies in between periods \( t \) and \( t + 1 \) with probability \( p \), in which case he gets 0 utility.

(a) Take \( n = 1 \), and find the optimal consumption stream \( c^* \) with \( c_0^* + c_1^* \leq w_0 \).
(b) Take \( n = \infty \), and find the optimal consumption stream \( c^* \) with \( c_0^* + c_1^* + \cdots \leq w_0 \).
(c) What would be your answer to part (b) if \( \rho = 1 \)?
(d) Solve part (c), assuming instead that Bob can get \( r_t \) from each dollars saved at \( t \), i.e., \( w \) dollars saved at \( t \) becomes \( wr_t \) dollars at \( t + 1 \), where \( (r_t) \) is i.i.d. with \( r_t > 0 \) and \( \delta E[\log r_t] \in (0, 1) \).

3. For any real-valued random variables \( X \) and \( Y \) and any increasing function \( g : \mathbb{R} \to \mathbb{R} \), prove or disprove the following statements.

(a) If \( X \) first-order stochastically dominates \( Y \), then \( g(X) \) first-order stochastically dominates \( g(Y) \).
(b) If \( X \) second-order stochastically dominates \( Y \), then \( g(X) \) second-order stochastically dominates \( g(Y) \).
(c) If \( X \) first-order stochastically dominates \( Y \), then \( X \) first-order stochastically dominates \( \alpha X + (1 - \alpha) Y \) for every \( \alpha \in [0, 1] \).

4. Ann has constant absolute risk aversion \( \alpha > 0 \) and initial wealth \( w \). She can buy shares from two divisible assets that are sold at unit price. One of assets pays a dividend \( X \sim N(2\mu, \sigma^2) \) and the other pays a dividend \( Y \sim N(\mu, \sigma^2) \) where \( X \) and \( Y \) are independently distributed and \( \mu > 1 \). She can buy any amount of shares from each asset, and she can keep some of her initial wealth in cash. Find the optimal portfolio for Ann.