This problem set reviews your knowledge of multivariate regression analysis. It requires you to answer questions related to the article on the reading list "Evidence of a shift in the short-run price elasticity of gasoline demand" by J. Hughes, C. Knittel and D. Sperling. Some of you may also find the article "An introduction to regression analysis" by A. Sykes, also on the reading list, is a useful reference for the material.

1. (60) In section 2 the authors estimate the following demand equation:

\[ \ln G_{jt} = \beta_0 + \beta_1 \ln P_{jt} + \beta_2 \ln Y_{jt} + \varepsilon_j + \varepsilon_{jt} \]

where \( G_{jt} \) is per capita gasoline consumption in gallons in month \( j \) and year \( t \), \( P_{jt} \) is the real retail price of gasoline in month \( j \) and year \( t \), \( Y_{jt} \) is real per capita disposable income in month \( j \) and year \( t \), \( \varepsilon_j \) represents unobserved demand factors that vary at the month level and \( \varepsilon_{jt} \) is a mean zero error term.

a) (10) What have the authors assumed about the price elasticity of demand when they wrote down the demand equation in this form? Remember the price elasticity of demand \( E_p = \frac{\partial G}{\partial P} \frac{P}{G} \).

b) (10) Go to table 1 in the appendix, now assuming the authors have obtained unbiased estimates of the parameters \( \beta_0, \beta_1 \) and \( \beta_2 \) what do they mean? (eg. the coefficient \( \beta_1 \) is -0.335 in the period 1975-1980, this represents...)
c) (10) Interpret the values of the monthly unobserved demand factors ($\varepsilon_j$)? What are these relative to? What can you say about the yearly pattern of gasoline demand from these coefficients?

d) (10) From the information presented in this table calculate the appropriate t-statistics for each of the $\beta$’s to test if it is different from 0. You will need the standard errors for each coefficient which are presented in brackets below the respective coefficient value in the table. For instance the standard error for the coefficient $\beta_1$ in the period 1975-1980 is 0.024.

e) (10) What do the *** next to some of the entries in the table indicate? How are they related to the t-statistics you calculated?

f) (10) The table presents the adjusted R-squared statistic for the two regressions. What does this number mean? If we calculated the unadjusted R-squared values, can we say whether these are larger or smaller than the adjusted R-squared values of 0.84 and 0.94 in this table?

2. (30) In table 2 and table 3 in the appendix, two alternate specifications for the demand equation are compared with the original double-log model.

a) (10) Under the linear specification for the period 1975-1980 the coefficient on the Price variable is -7.252. What is the implied elasticity of demand, assuming the linear model, if during a June month per capita demand was 40 gallons, and price was $1.70?

b) (10) What is the implied elasticity of demand if during July demand is 5 gallons higher (due to a month specific effect) and price is the same?

c) (10) Under the linear demand specification the demand elasticity varies within each period. Therefore the authors calculate an "average" elasticity of demand across each period. Do you think a time weighted or quantity weighted average is more reasonable and why?

3. (20) In section 3.3 the authors’ specify a model where there is an interaction between price and income

$$\ln G_{jt} = \beta_0 + \beta_1 \ln P_{jt} + \beta_2 \ln Y_{jt} + \beta_3 \ln P_{jt} \ln Y_{jt} + \varepsilon_j + \varepsilon_{jt}$$
where $\beta_3$ is the coefficient on the interaction term.

**a)** (10) Derive the price elasticity of demand using this model specification. You should get $E_p = \beta_1 + \beta_3 \ln Y_{jt}$. What would be the expression for the income elasticity of demand?

**b)** (10) Interpret the values of $\beta_3$ presented in table 7 in the appendix. What do these tell you about the effect of income on the price elasticity of demand? How has this changed from the 1975-80 period to today?

4. (40) (14.444 students only) In section 3.2 the author’s find that when the simultaneity bias is accounted for in the period 2001-2006 the estimate for the elasticity of demand changes from -0.042 to -0.077 and this change is statistically significant. The authors’ conclude that this is encouraging and that the effects of simultaneity are small relative to other factors. Give a brief (1/2 a page max.) critique of the authors’ methodology for addressing the simultaneity bias.