Dielectrics and Dipoles

Reading - Shen and Kong - Ch. 10

Outline

• Polarization and Dipole Density
• Dielectric Constant
• Microphones
• More Dielectric Actuators
True or False?

1. The magnetic moment is defined as
\[ m = i \alpha \]
and has units of Amp-m²

2. In a linear magnetic material, the magnetization is given by
\[ \vec{M} = \chi_m \vec{H} \]
where \( \chi_m \) is the magnetic susceptibility with units of m³

3. The energy stored in the magnetic field is
\[ \left( \frac{1}{2} \mu_0 \vec{H}^2 \right) \cdot \text{(Volume)} \]
and has units of Joules.
**Electric Fields**

\[ \oint_S \epsilon_0 \vec{E} \cdot d\vec{A} = \int_V \rho dV = Q_{\text{enclosed}} \]

**Magnetic Fields**

\[ \oint_S \vec{B} \cdot d\vec{A} = 0 \]

\[ \oint_C \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \left( \oint_S \vec{B} \cdot d\vec{A} \right) \]

\[ \oint_C \vec{H} \cdot d\vec{l} = \oint_S \vec{J} \cdot d\vec{A} + \frac{d}{dt} \int_S \epsilon \vec{E} \cdot d\vec{A} \]
Air Capacitors

\[ C v = q \]

\[ \epsilon_o \frac{A}{d} v = \sigma A \]

\[ E = \frac{\sigma}{\epsilon_o} \]

\[ \int_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho dV \]

\[ v = - \int \vec{E} \cdot d\vec{l} \]
... let’s insert a metal sheet between capacitor plates

- Capacitance increases
- Since $\sigma$ remained the same then $\nu$ decreases
... let’s insert an insulator sheet between capacitor plates

- Capacitance increases
- Since $\sigma$ remained the same then $\nu$ decreases

$\sigma \geq \sigma_{\text{dielectric}}$
Why is there $\sigma_{\text{dielectric}}$?

What is the magnitude of $\sigma_{\text{dielectric}}$?

When molecules are in an electric field they get stretched.

Electric field polarizes molecules ...
... turns them into Dipoles.
**What is a Dipole?**

\[ + \quad - \]

- **NO external E filed**

- **external E filed stretches the atom/molecule**

- **small amount of charge moved by field E**

\[ \vec{p} = \delta \vec{x} \]

**Dipole Moment**

**electric dipole moment,** \( p \) (or **electric dipole** for short), is a measure of the polarity of a system of electric charges. Here \( x \) is the displacement vector pointing from the negative charge to the positive charge. This implies that the electric dipole moment vector points from the negative charge to the positive charge. Note that the electric field lines run away from the positive charge and toward the negative charge. There is no inconsistency here, because the electric dipole moment has to do with the positions of the charges, not the field lines.
Analogy Between Magnetic and Electric Dipoles

**Magnetic Fields**

\[ \mathbf{B} \]

Magnetic Moment

\[ m = i \mathbf{a} \]

**Electric Field**

**Electric Dipole Moment**

\[ m = q \mathbf{d} \]
Superposition

The *magnetization* or *net magnetic dipole moment density* is given by

\[ \vec{M} = N \vec{m} \]

Number of dipoles per unit volume \([\text{m}^{-3}]\)

average magnetic dipole moment \([\text{A m}^2]\)

The *polarization* or *net electric dipole moment density* is given by

\[ \vec{P} = N \vec{p} \]

Number of dipoles per unit volume \([\text{m}^{-3}]\)

average electric dipole moment \([\text{C m}]\)
**Induced Magnetization**

For some materials, the *net magnetic dipole moment per unit volume* is proportional to the *H* field

\[
\overline{M} = \chi_m \overline{H}
\]

**MAGNETIC SUSCEPTIBILITY**

(dimENSIONLESS)

The effect of an applied magnetic field on a *magnetic* material is to create a net magnetic dipole moment per unit volume \(\overline{M}\)
Electric field polarizes molecules...

\[ \vec{P} = N \vec{p} \]
\[ = N \delta \vec{x} = N \alpha \vec{E} \]

polarization density, \( P \) (a.k.a electric polarization, or simply polarization) - density of permanent or induced electric dipole moments in a dielectric material. The SI unit of measure is coulombs per square meter.

Density of dipoles...

Electric field polarizes molecules...

\[ \vec{P} = \epsilon_0 \chi_e \vec{E} \]

electric susceptibility of a dielectric material is a measure of how easily it polarizes in response to an electric field.
Origin of the Dielectric Response

Polarizability, $\alpha$, tells us how easy is to disturb charge distribution, like the electron cloud of an atom or molecule, from its normal shape by an external electric field.

\[ \vec{P} = N \vec{p} = N \alpha \vec{E} \]

1. Electronic Polarizability ($\alpha_e$)
   Polarization of localized electrons

2. Ionic Polarizability ($\alpha_i$)
   Displacement of ions

3. Dipolar Polarizability ($\alpha_d$)
   Reorientation of polar molecules

4. Space Charge Polarizability ($\alpha_s$)
   Long range charge migration

Polarizability ($\alpha$) increases
Response Time Increases (slower response)
Bound Charges and Polarization

\[ \rho_{\text{bound}} = -\nabla \cdot \vec{P} \]

low dipole density

++-+-+-+-
+-+-+-
+-+-+-
+-+-+-

high dipole density

++-+-+-+-+
+-+-+-+
+-+-+-+
+-+-+-+

negative interface charge
... let's insert an insulator sheet between capacitor plates

\[ \int_S \varepsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho_{\text{total}} dV \]

\[ = \int_V (\rho_{\text{bound}} + \rho_{\text{free}}) dV \]

\[ = \int_V \left( -\nabla \cdot \vec{P} + \rho_{\text{free}} \right) dV \]

equivalently ...

\[ \int_S \left( \varepsilon_o \vec{E} + \vec{P} \right) \cdot d\vec{A} = \int_V \rho_{\text{free}} dV \]
... let's insert an insulator sheet between capacitor plates

Electric displacement \( D \)

\[
\vec{D} = \varepsilon_o \vec{E} + \vec{P} \\
= \varepsilon_o \vec{E} + \varepsilon_o \chi_e \vec{E} \\
= \varepsilon_o (1 + \chi_e) \vec{E} \\
\vec{D} = \varepsilon_o \varepsilon_r \vec{E} = \varepsilon \vec{E}
\]
where we define electric displacement field, $D$, as

$$\int_S \vec{D} \cdot d\vec{A} = \int_S \left( \epsilon_o \vec{E} + \vec{P} \right) \cdot d\vec{A} = \int_V \rho_{\text{free}} dV$$

Displacement field $D$ accounts for the effects of unbound (“free”) charges within materials.

Electric field $E$ accounts for the effects of total charges (both “bound” and “free”) within materials.

$$\int_S \epsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho_{\text{total}} dV$$
Ferroelectrics

A ferroelectric material develops a spontaneous polarization (builds up a charge) in response to an external electric field

- The polarization does not go away when the external field is removed
- The direction of the polarization is reversible
- Examples: BaTiO$_3$, PbTiO$_3$

Applications of Ferroelectric Materials

- Non-volatile FRAM (Ferroelectric Random Access Memory)
Why Worry About Dielectrics?

As transistors scale, insulation within the capacitor has become leaky...

Image by MIT OpenCourseWare.

Capacitor in 90, 65nm

SiO$_2$ $\varepsilon/\varepsilon_0 = 3.9$

Capacitor in 45nm

HfO$_2$ $\varepsilon/\varepsilon_0 = 25$
Energy Density of the Electric Field

What is the energy density stored in the capacitor?

For a capacitor with large, flat plates...

\[
\frac{W_S}{V} = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2} \frac{\varepsilon_0 \varepsilon_r A}{d} \nu^2
\]

\[
= \frac{1}{2} \frac{\varepsilon_0 \varepsilon_r A}{d} E^2 d^2 = \frac{1}{2} \vec{D} \cdot \vec{E}
\]

The electric field is not just the origin of electrostatic forces but also tells us about the stored energy!
Linear Dielectric Slab Actuator

Note: We are going to keep $Q$ constant on the plates.

\[ W_s = \frac{Cv^2}{2} = \frac{Q^2}{2C} \]

\[ f = - \left. \frac{\partial W_s}{\partial z} \right|_Q \]

\[ f = \frac{Q^2}{2C^2} \frac{dC}{dz} \]

\[ C(z) = \frac{\varepsilon_o \varepsilon_r z W}{s} + \frac{\varepsilon_o (L - z) W}{s} \]

\[ f = \]

21
Electromagnetic Energy Storage

Remember ...

**Magnetic**

\[
\frac{W_S}{V} = \frac{1}{2} \mu \vec{H} \cdot \vec{H} = \frac{1}{2} \vec{B} \cdot \vec{H}
\]

**Electric**

\[
\frac{W_S}{V} = \frac{1}{2} \epsilon \vec{E} \cdot \vec{E} = \frac{1}{2} \vec{D} \cdot \vec{E}
\]

**Magnetic machine**

**Electric machine**
Electric displacement

\[ \vec{D} = \varepsilon_o \vec{E} + \vec{P} \]

\[ = \varepsilon_o \vec{E} + \varepsilon_o \chi_e \vec{E} \]

\[ = \varepsilon_o (1 + \chi_e) \vec{E} \]

\[ \vec{D} = \varepsilon_o \varepsilon_r \vec{E} = \varepsilon \vec{E} \]

\[ \int_S \varepsilon_o \vec{E} \cdot d\vec{A} = \int_V \rho_{\text{total}} dV \]

\[ \int_S \vec{D} \cdot d\vec{A} = \int_V \rho_{\text{free}} dV \]

Magnetic

\[ \frac{W_S}{V} = \frac{1}{2} \mu \vec{H} \cdot \vec{H} \]

\[ = \frac{1}{2} \vec{B} \cdot \vec{H} \]

Electric

\[ \frac{W_S}{V} = \frac{1}{2} \varepsilon \vec{E} \cdot \vec{E} \]

\[ = \frac{1}{2} \vec{D} \cdot \vec{E} \]