Microscopic Ohm’s Law

Outline

Semiconductor Review
Electron Scattering and Effective Mass
Microscopic Derivation of Ohm’s Law
1. Judging from the filled bands, material A is an insulator.

2. Shining light on a semiconductor should decrease its resistance.

3. The band gap is a certain location in a semiconductor that electrons are forbidden to enter.
1-D Lattice of Atoms
Single orbital, single atom basis

Adding atoms...
• reduces curvature of lowest energy state (incrementally)
  • increases number of states (nodes)
• beyond ~10 atoms the bandwidth does not change with crystal size

Decreasing distance between atoms (lattice constant) ...
• increases bandwidth
From Molecules to Solids

N-1 nodes

0 nodes

Closely spaced energy levels form a “band” of energies between the max and min energies

N atoms → N states

4
Electron Wavepacket in Periodic Potential

For smooth motion
- wavepacket width $>>$ atomic spacing
- any change in lattice periodicity ‘scatters’ wavepacket
  - vibrations
  - impurities (dopants)
**Equivalent Free Particle**

Electron wavepacket

Coulomb potential due to nuclei

Effective ‘free’ electron wavepacket

Wavepacket moves as if it had an effective mass...

\[ F_{ext} = m \times a \]

Electron responds to external force as if it had an effective mass
**Surprise: Effective Mass for Semiconductors**

Electrons wavepackets often have effective mass smaller than free electrons!

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Germanium</th>
<th>Silicon</th>
<th>Gallium Arseneide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest energy bandgap at 300 K</td>
<td>$E_g$ (eV)</td>
<td>0.66</td>
<td>1.12</td>
<td>1.424</td>
</tr>
<tr>
<td>Effective mass for conductivity calculations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrons</td>
<td>$m_e^{*}\text{,cond}/m_0$</td>
<td>0.12</td>
<td>0.26</td>
<td>0.067</td>
</tr>
<tr>
<td>Holes</td>
<td>$m_h^{*}\text{,cond}/m_0$</td>
<td>0.21</td>
<td>0.36</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Which material will make faster transistors?
Approximate Wavefunction for 1-D Lattice
Single orbital, single atom basis

$k = 0$

$k ≠ 0$

$k = \pi/a$

$k$ is a convenient way to enumerate the different energy levels (count the nodes)

Bloch Functions: $\psi_{n,k}(r) = u_{n,k}(r)e^{ikr}$ $u_{n,k}(r) \approx$ orbitals
Energy Band for 1-D Lattice
Single orbital, single atom basis

- Number of states in band = number of atoms
- Number of electrons to fill band = number of atoms x 2 (spin)
From Molecules to Solids

The total number of states = \((\text{number of atoms}) \times (\text{number of orbitals in each atom})\)

**Bands of allowed energies for electrons**

**Bands Gap** - range of energy where there are no "allowed states"

The total number of states = (number of atoms) \(\times\) (number of orbitals in each atom)
Bands from Multiple Orbitals

Atom → Solid

$\begin{align*}
& n = 3 \\
& n = 2 \\
& n = 1
\end{align*}$

- Each atomic state $\rightarrow$ a band of states in the crystal
- These are the “allowed” states for electrons in the crystal
- Fill according to Pauli Exclusion Principle
- There may be gaps between the bands
- These are “forbidden” energies where there are no states for electrons

These two facts are the basis for our understanding of metals, semiconductors, and insulators !!!

Example of Na

$Z = 11 \quad 1s^22s^22p^63s^1$

What do you expect to be a metal ?

What about semiconductors like silicon?

Fill the Bloch states according to Pauli Principle.

\[
\begin{array}{c}
Z = 14 \\
1s^2 2s^2 2p^6 3s^2 3p^2
\end{array}
\]

Total # atoms = N
Total # electrons = 14N

It appears that, like Na, Si will also have a half filled band: The 3s3p band has 4N orbital states and 4N electrons.

By this analysis, Si should be a good metal, just like Na.

But something special happens for Group IV elements.
Antibonding states

Bonding states

4N states

N states

1s, 2s, 2p, 3s, 3p

2N electrons fill these states

8N electrons fill these states

Z = 14

1s²2s²2p⁶3s²3p²

Total # atoms = N

Total # electrons = 14N

The 3s-3p band splits into two:

Antibonding states

Bonding states
Controlling Conductivity: Doping Solids

ACCEPTOR DOPING:  
P-type Semiconductor  
Dopants: B, Al

DONOR DOPING  
N-type Semiconductor  
Dopants: As, P, Sb
Making Silicon Conduct

- Metal
- Insulator or Semiconductor $T=0$
- Semi-Conductor $T \neq 0$
- n-Doped Semi-Conductor
Today’s Culture Moment

The bandgap in Si is 1.12 eV at room temperature. What is “reddest” color (the longest wavelength) that you could use to excite an electron to the conduction band?
Semiconductor Resistor

Given that you are applying a constant E-field (Voltage) why do you get a fixed velocity (Current)? In other words why is the Force proportional to Velocity?

How does the resistance depend on geometry?
**Microscopic Scattering**

A local, unexpected change in $V(x)$ of electron as it approaches the impurity

Strained region by impurity exerts a scattering force

Scattering from thermal vibrations
Balance equation for forces on electrons:

\[
m \frac{d\mathbf{v}(r, t)}{dt} = -m \frac{\mathbf{v}(r, t)}{\tau} - e \left[ \mathbf{E}(r, t) + \mathbf{v}(r, t) \times \mathbf{B}(r, t) \right]
\]

Drag Force

Lorentz Force
Microscopic Variables for Electrical Transport

Drude Theory

Balance equation for forces on electrons:

\[ m \frac{d\mathbf{v}(r, t)}{dt} = -m \frac{\mathbf{v}(r, t)}{\tau} - e [\mathbf{E}(r, t) + \mathbf{v}(r, t) \times \mathbf{B}(r, t)] \]

Drag Force \hspace{2cm} Lorentz Force

In steady-state when B=0:

*Note: Inside a semiconductor \( m = m^* \) (effective mass of the electron)*

\[ \mathbf{v} = -\frac{e\tau}{m^*} \mathbf{E}_{DC} \]

\[ \mathbf{J} = -ne\mathbf{v} = \frac{ne^2\tau}{m^*} \mathbf{E}_{DC} \]

\[ \mathbf{J} = \sigma \mathbf{E}_{DC} \quad \text{and} \quad \sigma = \frac{ne^2\tau}{m^*} \]
Semiconductor Resistor

\[ \vec{J} = \sigma \vec{E}_{DC} \quad \text{and} \quad \sigma = \frac{ne^2\tau}{m^*} \]

Recovering macroscopic variables:

\[ I = \int \vec{J} \cdot d\vec{A} = \sigma \int \vec{E} \cdot d\vec{A} = \sigma \frac{V}{l} A \]

\[ V = I \frac{l}{\sigma A} = I \frac{\rho l}{A} = IR \]

OHM’s LAW
Microscopic Variables for Electrical Transport

\[ \mathbf{J} = \sigma \mathbf{E}_{DC} \quad \text{and} \quad \sigma = \frac{ne^2 \tau}{m^*} \]

\[ \tau = \frac{m^* \sigma}{ne^2} \]

For silicon crystal doped at \( n = 10^{17} \text{ cm}^{-3} \):
\( \sigma = 11.2 \text{ (}\Omega \text{ cm})^{-1} \), \( \mu = 700 \text{ cm}^2/(\text{Vs}) \) and \( m^* = 0.26 \text{ m}_o \)

\[ \tau = \frac{0.26(9.1 \times 10^{-31} \text{ kg})(11.200 \text{ m}^{-1}\Omega^{-1})}{10^{23} \text{ m}^{-3}(1.6 \times 10^{-19} \text{ C})^2} = 10^{-13} \text{ s} = 100 \text{ fs} \]

At electric fields of \( E = 10^6 \text{ V/m} = 10^4 \text{ V/cm}, \)
\( \nu = \mu E = 700 \text{ cm}^2/(\text{Vs}) \times 10^4 \text{ V/cm} = 7 \times 10^6 \text{ cm/s} = 7 \times 10^4 \text{ m/s} \)
scattering event every 7 nm \( \sim 25 \) atomic sites
Electron Mobility

Electron wavepacket

$\vec{J} = \sigma \vec{E} = ne \vec{v} = ne \mu \vec{E}$

$\vec{v} = \mu \vec{E}$

Electron velocity for a fixed applied E-field

Energy

Conduction Band

Electron

$E_C$

Valence Band

Hole

$E_V$
**Electron Mobility**

\[ \sigma_e = n |e| \mu_e = 1/\rho \]

- **Intrinsic Semiconductors** (no dopants)
  - Dominated by number of carriers, which increases exponentially with increasing temperature due to increased probability of electrons jumping across the band gap
  - At high enough temperatures phonon scattering dominates \( \rightarrow \) velocity saturation

- **Metals**
  - Dominated by mobility, which decreases with increasing temperature
Key Takeaways

Electron wavepacket

Coulomb potential due to nuclei

Wavepacket moves as if it had an effective mass...

\[ F_{\text{ext}} = m \cdot a \]

\[ J = -n e \bar{v} = \frac{n e^2 \tau}{m^*} \vec{E}_{DC} \]