Announcements
First Hour Exam - Oct. 7, 7:30-9:30 pm; thru 10/2/09, PS #4

Review/Diode model wrap-up
Exponential diode: \( i_D(v_{AB}) = I_S \left( e^{qv_{AB}/kT} - 1 \right) \) (holes) (electrons)
with \( I_S = A q n_i^2 \left[ (D_h/N_{Dn} w_{n^*}) + (D_e/N_{Ap} w_{p^*}) \right] \)
Observations: Saturation current, \( I_S \), goes down as doping levels go up
Injection is predominantly into more lightly doped side
Asymmetrical diodes: the action is on the lightly doped side
Diffusion charge stores; diffusion capacitance: (Recitation topic)
Excess carriers in quasi-neutral region = Stored charge

Bipolar junction transistor operation and modeling
Bipolar junction transistor structure
Qualitative description of operation: 1. Visualizing the carrier fluxes
(using npn as the example) 2. The control function
3. Design objectives
Operation in forward active region, \( v_{BE} > 0, v_{BC} < 0 \): \( \delta_E, \delta_B, \beta_F, I_{ES} \)
**Biased p-n junctions: current flow, cont.**

**The saturation current of three diode types:**

$I_s$'s dependence on the relative sizes of $w$ and $L_{\text{min}}$

**Short-base diode, $w_n << L_h, w_p << L_e$:**

$$J_h(x_n) = q \frac{n_i^2}{N_{Dn}} \frac{D_h}{(w_n - x_n)} \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

$$J_e(-x_p) = q \frac{n_i^2}{N_{Ap}} \frac{D_e}{(w_p - x_p)} \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

$$i_D = Aq n_i^2 \left[ \frac{D_h}{N_{Dn}(w_n - x_n)} + \frac{D_e}{N_{Ap}(w_p - x_p)} \right] \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

**Long-base diode, $w_n >> L_h, w_p >> L_e$:**

$$J_h(x_n) = q \frac{n_i^2}{N_{Dn}} \frac{D_h}{L_h} \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

$$J_e(-x_p) = q \frac{n_i^2}{N_{Ap}} \frac{D_e}{L_e} \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

$$i_D = Aq n_i^2 \left[ \frac{D_h}{N_{Dn} L_h} + \frac{D_e}{N_{Ap} L_e} \right] \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

**General diode:**

$$i_D = Aq n_i^2 \left[ \frac{D_h}{N_{Dn} w_{n,\text{eff}}} + \frac{D_e}{N_{Ap} w_{p,\text{eff}}} \right] \left[ e^{\frac{qV_{AB}}{kT}} - 1 \right]$$

Hole injection into n-side  \hspace{1cm} Electron injection into p-side

Note: $w_{n,\text{eff}} = L_h \tanh(w_n - x_n)$, $w_{p,\text{eff}} = L_e \tanh(w_p - x_p)$
Asymmetrically doped junctions: an important special case

Current flow impact/issues

A p+-n junction ($N_{Ap} >> N_{Dn}$):

$$i_D = A q n_i^2 \left[ \frac{D_h}{N_{Dn} w_{n,eff}} + \frac{D_e}{N_{Ap} w_{p,eff}} \right] \left[ e^{\frac{q V_{AB}}{kT}} - 1 \right] \approx A q n_i^2 \frac{D_h}{N_{Dn} w_{n,eff}} \left[ e^{\frac{q V_{AB}}{kT}} - 1 \right]$$

Hole injection into n-side

An n+-p junction ($N_{Dn} >> N_{Ap}$):

$$i_D = A q n_i^2 \left[ \frac{D_h}{N_{Dn} w_{n,eff}} + \frac{D_e}{N_{Ap} w_{p,eff}} \right] \left[ e^{\frac{q V_{AB}}{kT}} - 1 \right] \approx A q n_i^2 \frac{D_e}{N_{Ap} w_{p,eff}} \left[ e^{\frac{q V_{AB}}{kT}} - 1 \right]$$

Electron injection into p-side

Note that in both cases the minority carrier injection is predominately into the lightly doped side.

Note also that it is the doping level of the more lightly doped junction that determines the magnitude of the current, and as the doping level on the lightly doped side decreases, the magnitude of the current increases.

Two very important and useful observations!!
**Biased p-n junctions:** excess minority carrier (diffusion) charge stores

**Diffusion charge store, and diffusion capacitance:**

*Using example of asymmetrically doped p-n diode*

Notice that the stored positive charge (the excess holes) and the stored negative charge (the excess electrons) occupy the same volume in space (between \( x = x_n \) and \( x = w_n \))!

\[
q_{A,DF}(v_{AB}) = A q \left[ p'(x_n) - p'(w_n) \right] \left[ \frac{w_n - x_n}{2} \right] \approx A q \frac{n_i^2}{N_{Dn}} \left[ e^{qv_{AB}/kT} - 1 \right] \frac{w_{n,\text{eff}}}{2}
\]

The charge stored depends non-linearly on \( v_{AB} \). As we did in the case of the depletion charge store, we define an incremental linear equivalent diffusion capacitance, \( C_{df}(V_{AB}) \), as:

\[
C_{df}(V_{AB}) \equiv \frac{\partial q_{A,DF}}{\partial v_{AB}} \bigg|_{v_{AB}=V_{AB}} \approx A \frac{q^2}{2kT} w_{n,\text{eff}} \frac{n_i^2}{N_{Dn}} e^{qv_{AB}/kT}
\]
Diffusion capacitance, cont.:

A very useful way to write the diffusion capacitance is in terms of the bias current, $I_D$:

$$I_D \approx A q n_i^2 \frac{D_h}{N_{Dn} w_{n,eff}} \left[ e^{q V_{AB} / kT} - 1 \right] \approx A q n_i^2 \frac{D_h}{N_{Dn} w_{n,eff}} e^{q V_{AB} / kT} \quad \text{for} \quad V_{AB} >> kT$$

To do this, first divide $C_{df}$ by $I_D$ to get:

$$\frac{C_{df}(V_{AB})}{I_D(V_{AB})} \approx \frac{A}{2kT} \frac{q^2}{N_{Dn}} \frac{w_{n,eff}^2}{N_{Dn}} e^{q V_{AB} / kT} = \frac{q w_{n,eff}^2}{2kT D_h}$$

Isolating $C_{df}$, we have:

$$C_{df}(V_{AB}) \approx \frac{w_{n,eff}^2}{2 D_h} \frac{q I_D(V_{AB})}{kT}$$

* Notice that the area of the device, $A$, does not appear explicitly in this expression. Only the total current!
Comparing charge stores; small-signal linear equivalent capacitors:

**Parallel plate capacitor**

\[ q_{A,PP} = A \frac{\varepsilon}{d} v_{AB} \]

\[ C_{pp}(V_{AB}) = \frac{\partial q_{A,PP}}{\partial v_{AB}} \bigg|_{v_{AB}=V_{AB}} = A \frac{\varepsilon}{d} \]

**Depletion region charge store**

\[ q_{A,DP}(v_{AB}) = -A \sqrt{2q \varepsilon_{Si} [\phi_b - v_{AB}] \frac{N_{Ap} N_{Dn}}{N_{Ap} + N_{Dn}}} \]

\[ C_{dp}(V_{AB}) = A \sqrt{\frac{q \varepsilon_{Si}}{2[\phi_b - V_{AB}]} \frac{N_{Ap} N_{Dn}}{N_{Ap} + N_{Dn}}} = A \frac{\varepsilon_{Si}}{w(V_{AB})} \]

**QNR region diffusion charge store**

Note: Approximate because we are only accounting for the charge store on the lightly doped side.

\[ q_{AB,DF}(v_{AB}) \approx A q_n^2 \frac{D_h}{N_{Dn} w_{n,eff}} \left[ e^{q v_{AB}/kT} - 1 \right] \]

\[ C_{df}(V_{AB}) \approx \frac{w_{n,eff}^2}{2 D_h} \frac{q I_D(V_{AB})}{kT} \]

Clif Fonstad, 10/1/09

Lecture 7 - Slide 6
p-n diode: large signal model including charge stores

$q_{AB}$: Excess carriers on p-side + excess carriers on n-side + junction depletion charge.

small signal linear equivalent circuit

\[ g_d = \left. \frac{\partial i_D}{\partial v_{AB}} \right|_{v_{AB}=V_{AB}} \approx \begin{cases} 0 & \text{for } V_{AB} < 0 \\ \frac{q I_D}{kT} & \text{for } V_{AB} >> \frac{kT}{q} \end{cases} \]

\[ C_d (V_{AB}) = \left. \frac{\partial q_{AB}}{\partial v_{AB}} \right|_{v_{AB}=V_{AB}} \approx \begin{cases} C_{dp} (V_{AB}) & \text{for } V_{AB} < 0 \\ C_{dp} (V_{AB}) + C_{df} (V_{AB}) & \text{for } V_{AB} >> \frac{kT}{q} \end{cases} \]
Moving on to **transistors**!

**Amplifiers/Inverters:** back to 6.002

An MOS amplifier or inverter: the transistor is an n-channel MOSFET

A bipolar amplifier or inverter: the transistor is an npn BJT

\[
v_T(t) = V_T + v_t(t)
\]

\[
V_{CC} v_{OUT} + - v_{IN} + - R_T D S G R_D
\]

Clif Fonstad, 10/1/09

Lecture 7 - Slide 8
**n pn BJT:** Connecting with the n-channel MOSFET from 6.002  
A very similar behavior*, and very similar uses.

*At its output each device looks like a current source controlled by the input signal.
How do we make a BJT?

Basic Bipolar Junction Transistor (BJT) - *cross-section*

An npn BJT
Adapted from Fig. 8.1 in Text

The heart of the device, and what we will model

How does it work?
**Bipolar Junction Transistors:** basic operation and modeling…

... how the base-emitter voltage, $v_{BE}$, controls the collector current, $i_C$

Reverse biased $v_{CB}$, the reverse bias on the collector-base junction, insures collection of those electrons injected across the E-B junction that reach the C-B junction as the collector current, $i_C$.

Forward biased $v_{BE}$, the bias on the emitter-base junction, controls the injection of electrons across the E-B junction into the base and toward the collector.

A good way to envision this is to think "carrier fluxes":

*Next foil*
**Bipolar Junction Transistors:** the carrier fluxes through an npn

The base supplies the small hole flux

Reverse biased BC junction collects electrons coming across the base from the emitter

Forward biased n⁺-p EB junction emits electrons into the base towards the BC junction

Our next task is to determine:

Given a structure, what are \( i_E(v_{BE},v_{CE}) \), \( i_C(v_{BE},v_{CE}) \), and \( i_B(v_{BE},v_{CE}) \)?
Bipolar Junction Transistors: basic operation and modeling...

... how the base-emitter voltage, $v_{BE}$, controls the collector current, $i_C$. 

Electron flux

Hole flux

Excess Carriers: 

$$n_i^2/N_{AB} (e^{qv_{BE}/kT} - 1)$$

$$n_i^2/N_{DE} (e^{qv_{BE}/kT} - 1)$$
**Bipolar Junction Transistors:** basic operation and modeling...

... how the base-emitter voltage, $v_{BE}$, controls the collector current, $i_C$

Excess Carriers:

This is rigorous for $v_{CB} = 0$, but also very good when $v_{CB} > 0$. 
**nnp BJT:** Forward active region operation, $v_{BE} > 0$ and $v_{BC} \leq 0$

**Excess Carriers:**

\[
\begin{align*}
(p', n') & \\
(n_i^2/N_{DE})(e^{qV_{BE}/kT} - 1) & \\
0 & \text{(ohmic)}
\end{align*}
\]

\[
\begin{align*}
&\sim 0 \quad (v_{BC} < 0) \\
&0 \quad \text{(ohmic)}
\end{align*}
\]

**Currents:**

\[
\begin{align*}
&i_e, i_h \\
i_{hE} & = \delta E i_e E \\
i_{eE} & \\
-i_C & = i_{eE} (1 - \delta_B) \\
i_{B} & = i_{hE} + \delta_B i_{eE} \\
&= i_{eE} (\delta E + \delta_B)
\end{align*}
\]
**npn BJT:** Approximate model for \(i_E(v_{BE},v_{BC})\) and \(i_C(v_{BE},v_{BC})\) in forward active region, \(v_{BE}>0, v_{BC}<0\)

The emitter current, \(i_E\)

Begin with the good current, the electron current into the base, \(i_{eE}\):

\[
i_{eE} = -A q n_i^2 \frac{D_e}{N_{AB} w_{B,eff}} \left[e^{\frac{q v_{BE}}{kT}} - 1\right]
\]

Next find the bad current, the hole current back into the emitter, \(i_{hE}\):

\[
i_{hE} = -A q n_i^2 \frac{D_h}{N_{DE} w_{E,eff}} \left[e^{\frac{q v_{BE}}{kT}} - 1\right]
\]

and write it as a fraction of \(i_{eE}\):

\[
i_{hE} = \frac{N_{AB} w_{B,eff}}{D_e} \frac{D_h}{N_{DE} w_{E,eff}} i_{eE} = \delta_E i_{eE}
\]

We'll define \(\delta_E\) on the next foil.
**nnpn BJT**: Approximate forward active region model, cont.

The emitter current, $i_E$, cont.

In writing the last equation we introduced the **emitter defect**, $\delta_E$:

$$
\delta_E = \frac{i_{hE}}{i_{eE}} = \frac{D_h}{D_e} \frac{N_{AB}}{N_{DE}} \frac{W_{B,eff}}{W_{E,eff}}
$$

To finish for now with the emitter current, we write it, $i_E$, in terms of the emitter electron current, $i_{eE}$:

$$
i_E = i_{eE} + i_{hE} = \left(1 + \frac{i_{hE}}{i_{eE}}\right) i_{eE} = (1 + \delta_E) i_{eE}
$$
**n,p,n BJT:** Approximate forward active region model, cont.

The collector current, $i_C$

The collector current is the electron current from the emitter, $i_{eE}$, minus the fraction that recombines in the base, $\delta_B i_{eE}$:

$$i_C = (1 - \delta_B) i_{eE}$$

To find the fraction that recombine, i.e. the base defect, $\delta_B$, we note that we can write the total recombination in the base, $\delta_B i_{eE}$, as:

$$\delta_B i_{eE} = -A q \int_{0}^{w_B} \frac{n'(x)}{\tau_{eB}} dx$$
**nnp BJT:** Approximate forward active region model, cont.

The base defect, $\delta_B$

If the recombination in the base is small (as it is in a good BJT) then the excess electron concentration will be nearly triangular and we can say:

$$\int_0^{w_B} n'(x) dx \approx \frac{n'(0) w_{B,\text{eff}}}{2}$$

and

$$i_{eE} \approx -A q D_{eB} \frac{n'(0)}{w_{B,\text{eff}}}$$

Thus

$$\delta_B = \frac{-A q \int_0^{w_B} n'(x) dx}{i_{eE}} \approx \frac{-A q n'(0) w_{B,\text{eff}}}{2 \tau_{eB}} = \frac{w_{B,\text{eff}}}{2 D_{eB} \tau_{eB}} = \frac{w_{B,\text{eff}}}{2 L_{eB}^2}$$

The collector current, $i_C$, cont.

Returning to the collector current, $i_C$, we now want to relate it to the total emitter current:

$$i_C = -(1 - \delta_B) i_{eE}$$

$$i_E = (1 + \delta_E) i_{eE}$$

$$i_C = -\frac{(1 - \delta_B)}{(1 + \delta_E)} i_E = -\alpha_F i_E$$

with

$$\alpha_F = \frac{(1 - \delta_B)}{(1 + \delta_E)}$$
nnpn BJT: Approximate forward active region model, cont.

So far...

We have:  
\[ i_E = -Aq n_i^2 \left( \frac{D_h}{N_{DE} w_{E,\text{eff}}} + \frac{D_e}{N_{AB} w_{B,\text{eff}}} \right) \left[ e^{qV_{BE}/kT} - 1 \right] \]

\[ = -I_{ES} \left[ e^{qV_{BE}/kT} - 1 \right] \quad \text{with} \quad I_{ES} = Aq n_i^2 \left( \frac{D_h}{N_{DE} w_{E,\text{eff}}} + \frac{D_e}{N_{AB} w_{B,\text{eff}}} \right) \]

\[ ... \text{and we have:} \]

\[ i_C \propto i_E : \quad i_C = -\alpha_F i_E \quad \text{with} \quad \alpha_F \equiv -\frac{i_C}{i_E} = \frac{(1 - \delta_B)}{(1 + \delta_E)} \]

These relationships can be represented by a simple circuit model:

\[ i_E = -i_F \quad \text{with} \quad i_F = I_{ES} \left( 1 - e^{qV_{BE}/kT} \right) \]

\[ i_C = \alpha_F i_F \quad \text{with} \quad \alpha_F \equiv -\frac{i_C}{i_E} = \frac{(1 - \delta_B)}{(1 + \delta_E)} \]

\[ i_B = -i_E - i_C = (1 - \alpha_F) i_F \]

Note:  
\[ i_F = -i_E. \]
**n-p-n BJT:** Approximate forward active region model, cont.

The base supplies the small hole flux.

Reverse biased BC junction collects electrons coming across the base from the emitter.

Forward biased n^-p EB junction emits electrons into the base towards the BC junction.

\[
i_E = -i_F = -I_{ES}\left(1 - e^{\frac{qV_{BE}}{kT}}\right)\]

\[
i_C = \alpha_F i_F\]

\[
i_B = -i_E - i_C = (1 - \alpha_F)i_F\]
nnp BJT: What our model tells us about device design.

We have:

\[ \alpha_F = \frac{(1 - \delta_B)}{(1 + \delta_E)} \]

and the defects, \( \delta_E \) and \( \delta_B \), are given by:

\[ \delta_E = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DE}} \cdot \frac{w_{B,eff}}{w_{E,eff}} \]

\[ \delta_B \approx \frac{w_{B,eff}^2}{2L_{eB}^2} \]

We want \( \alpha_F \) to be as close to one as possible, and clearly the smaller we can make the defects, the closer \( \alpha_F \) will be to one. Thus making the defects small is the essence of good BJT design:

Doping: npn with \( N_{DE} >> N_{AB} \)

\( w_{B,eff} \): very small

\( L_{eB} \): very large and \( >> w_{B,eff} \)
**nnp BJT:** Well designed structure (Large $\beta_F$, small $\delta_E$ and $d_B$)

$\delta_E$ and $\delta_B$ are small and $\alpha_F$ is $\approx$ 1 when $N_{DE} >> N_{AB}$, $w_E << L_{hE}$, $w_B << L_{eB}$

**Excess Carriers:**

\[
\frac{n_i^2}{N_{DE}} (e^{qV_{BE}/kT} - 1)
\]

\[
\frac{n_i^2}{N_{AB}} (e^{qV_{BE}/kT} - 1)
\]

\[0\] (ohmic) \[0\] (ohmic)

x

\[0\] (vBC = 0)

\[w_B\]

\[w_B + w_C\]

\[-w_E\]

\[p', n'\]

\[\alpha_F\]

**Currents:**

\[i_E \approx i_{eE}(1 + \delta_E)\]

\[i_{hE} = \delta_E i_{eE}\]

\[i_{eE}\]

\[-i_C \approx i_{eE}(1 - \delta_B)\]

\[-i_B \approx i_E - (-i_C) = i_{eE}(\delta_E + \delta_B) \approx i_{eE}\delta_E\]
nnpn BJT, cont.: more observations about F.A.R. model

It is very common to think of $i_B$, rather than $i_E$, as the controlling current in a BJT. In this case we write $i_C$ as depending on $i_B$:

$$i_E = -i_F = -I_{ES} \left[ e^{qV_{BE}/kT} - 1 \right]$$

$$i_C = \alpha_F i_F$$

$$i_B = (1 - \alpha_F) i_F$$

$$i_b = (1 - \alpha_F) i_F$$

$$i_E = -i_C - i_B = (\beta_F + 1) i_B$$

with \( \beta_F \equiv \frac{\alpha_F}{1 - \alpha_F} = \frac{(1 - \delta_B)}{\delta_E + \delta_B} \) and \( I_{BS} \equiv (1 - \alpha_F) I_{ES} = \frac{I_{ES}}{(\beta_F + 1)} \)

Two circuit models that fit this behavior are the following:

Note:

$$\alpha_F = \frac{\beta_F}{(\beta_F + 1)}$$
**nnpn BJT:** Equivalent FAR models

\[ I_{ES} = A q n_i^2 \left( D_h / N_D W_{E,Eff} + D_e / N_A W_{B,Eff} \right) \]
\[ \alpha_F = \frac{\beta_F}{(\beta_F + 1)} \]

\[ \beta_F = \frac{\alpha_F}{(1 - \alpha_F)} \]

\[ I_{BS} = I_{ES} / (\beta_F + 1) \]

A useful model using a break-point diode:

This is a very useful model to use when finding the bias point in a circuit.
nnp BJT: The Ebers-Moll model
The forward model is what we use most, but adding the reverse model we cover the entire range of possible operating conditions.

Forward:

\[ I_{ES} = Aq_n i^2 \left( \frac{D_h}{N_{DE} w_{E,eff}} + \frac{D_e}{N_{AB} w_{B,eff}} \right) \]

\[ \beta_F = \frac{(1-\delta_B)}{(\delta_E + \delta_B)} \]

\[ \alpha_F = \frac{(1-\delta_B)}{(1+\delta_E)} \]

\[ \delta_E \approx \frac{w_{B,eff}}{2D_e \tau_e} = \frac{w_{B,eff}}{2L_e^2} \]

Combined they form the full Ebers-Moll model:

Reverse:

\[ I_{CS} = Aq_n i^2 \left( \frac{D_h}{N_{DC} w_{C,eff}} + \frac{D_e}{N_{AB} w_{B,eff}} \right) \]

\[ \beta_R = \frac{(1-\delta_B)}{(\delta_C + \delta_B)} \]

\[ \alpha_R = \frac{(1-\delta_B)}{(1+\delta_C)} \]

\[ \delta_C \approx \frac{i_{BC}}{i_{EC}} = \frac{D_h}{D_e} \cdot \frac{N_{AB}}{N_{DC}} \cdot \frac{w_{B,eff}}{w_{C,eff}} \]

\[ \delta_B \approx \frac{w_{B,eff}}{2D_e \tau_e} = \frac{w_{B,eff}}{2L_e^2} \]

Note: \[ i_F = -i_E(v_{BE},0) \] and \[ i_R = -i_C(0,v_{BC}). \]
nnp BJT: The Gummel-Poon model

Another common model can be obtained from the Ebers-Moll model is the Gummel-Poon model:

Forward: $I_S = \frac{\beta_F}{(\beta_F + 1)} I_{ES} = \frac{\beta_R}{(\beta_R + 1)} I_{CS} = \alpha_F I_{ES} = \alpha_R I_{CS}$

Combined they form the Gummel-Poon model:

• Aside from the historical interest, another value this has for us in 6.012 is that it is an interesting exercise to show that the two forward circuits above are equivalent.

You even less responsible for this model.
• Review/Junction diode model wrap-up

Refer to "Lecture 6- Summary" for a good overview

**Diffusion capacitance:** adds to depletion capacitance

\[ C_{df}(V_{AB}) \approx (qI_D/kT)[(w_n-x_n)^2/D_h] \] 
(area doesn't enter expression!)

• Bipolar junction transistor operation and modeling

**Currents (forward active):** (npn example)

\[ i_E(v_{BE},0) = - I_{ES} \left( e^{qv_{BE}/kT} - 1 \right) \]

\[ i_C(v_{BE},0) = - \alpha_F i_E(v_{BE},0) \]

with \( \alpha_F \equiv [(1 - \delta_B)/(1 + \delta_E)] \)

Emitter defect, \( \delta_E \equiv (D_hN_{AB}w_B^*/D_eN_{DE}w_E^*) \)
(ratio of hole to electron current across E-B junction)

Base defect, \( \delta_B \equiv (w_B^2/2L_e^2) \)
(fraction of injected electrons recombining in base)

Also, \( i_B(v_{BE},0) = [(d_E + d_B)/(1 + d_E)] i_E(v_{BE},0) \)
and, \( i_C(v_{BE},0) = b_F i_B(v_{BE},0), \)
with \( b_F \equiv a_F/(1 - a_F) = [(1 - d_B)/(d_E + d_B)] \)