Suggested Reading Assignment: 5.2.1, 5.2.2, 9.2

Quiz 2 will be on Thursday, November 17 at 10-11 a.m. It will cover material through P. S. #8, with a focus on sinusoidal steady state and transient waves on transmission lines; parallel plate, rectangular, and dielectric waveguides. Quiz 2 Formula Sheets (as attached to this problem set) will be provided.

Problem 8.1
Switched transmission line systems with an initial dc voltage can be used to generate high voltage pulses of short time duration. The line shown is charged up to a dc voltage $V_0$ when at $t=0$ the load switch is closed and the source switch is opened.

(a) What are the initial line voltage and current at $t=0$? What are $V_+$ and $V_-$?

(b) Sketch the time dependence of the load voltage.


Problem 8.2

A transmission line of length $\ell$ and characteristic impedance $Z_0$ has a matched load at $z=0$ and is short circuited at $z=\ell$. The speed of electromagnetic waves on the transmission line is $c$ so that the one-way transit time is $T = \ell/c$. The transmission line is unexcited until at $t=0$ it is hit by lightning so that the line voltage and current at $t=0$ are

\[
\begin{align*}
  v(z, t = 0) &= V_0 & 0 < z < \ell \\
  i(z, t = 0) &= 0 & 0 < z < \ell
\end{align*}
\]
a) What are $V_+(t - z/c)$ and $V_-(t + z/c)$ at $t = 0$?

b) Plot the voltage and current at $z = 0$ as a function of time.

c) Plot the current at $z = \ell$ as a function of time.

d) Plot the voltage and current as a function of $z$ for $0 < z < \ell$ at time $t = \frac{T}{2}$.

**Problem 8.3**
An air-filled metal rectangular waveguide has cross-sectional area dimensions $a = 2$ cm and $b = 1$ cm.

a) What $TE_{mn}$ and $TM_{mn}$ mode have the lowest cut-off frequencies and what are these frequencies?

b) Over what frequency range will this waveguide operate at a single mode? What is the mode?

**Problem 8.4**

A parallel plate waveguide with spacing $d$, dielectric permittivity $\varepsilon$, and magnetic permeability $\mu$ supports $TE_n$ and $TM_n$ modes given by:

**$TE_n$**

$\vec{E} = E_0 \sin k_z z \sin(\omega t - k_x x) \hat{t}_y$

$\vec{H} = \frac{E_0}{\eta k} \left[ -k_z \cos k_z z \cos(\omega t - k_x x) \hat{t}_x + k_x \sin k_z z \sin(\omega t - k_x x) \hat{t}_z \right]$

**$TM_n$**

$\vec{E} = \frac{E_0}{k} \left[ k_z \sin k_z z \sin(\omega t - k_x x) \hat{t}_x - k_x \cos k_z z \cos(\omega t - k_x x) \hat{t}_z \right]$

$\vec{H} = \frac{E_0}{\eta} \cos k_z z \cos(\omega t - k_x x) \hat{t}_y$

$k_z = \frac{n\pi}{d}, \quad k_x = \sqrt{\frac{\omega^2 \varepsilon \mu - k_z^2}{d^2}}$
a) What are the surface charge densities on the $z=0$ and $z=d$ surfaces for $\text{TE}_n$ and $\text{TM}_n$ modes?

b) What are the surface current densities on the $z=0$ and $z=d$ surfaces for $\text{TE}_n$ and $\text{TM}_n$ modes?

c) Find the equation for the magnetic field lines that go through coordinates $(x_0, z_0)$ for the $\text{TE}_n$ modes at $t=0$ defined as:
\[
\frac{dz}{dx} = \frac{H_z}{H_x}
\]

Hint: \[\int \cot u \, du = \ln(\sin u) + \text{Constant}\]

d) Find the equation for the electric field lines that go through coordinates $(x_0, z_0)$ for the $\text{TM}_n$ modes at $t=0$ defined as:
\[
\frac{dz}{dx} = \frac{E_z}{E_x}
\]

Hint: \[\int \tan u \, du = -\ln(\cos u) + \text{Constant}\]

e) Extra Credit: Using your favorite drawing program, draw some field lines of parts (c) and (d) that illustrate the fundamental shape of the $\text{TE}_1$ and $\text{TM}_1$ modes.