Problem 8.1  A pair of two-level systems  Do problem 19.2 in the text.

Problem 8.2  Spin 1/2 Matrices

Consider the three Pauli spin matrices $\sigma_i$ and the identity matrix.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Argue that any two by two Hermitian matrix can be written as a linear combination of these four matrices. Consequently, write the Hamiltonian

$$\hat{H} = \begin{pmatrix} \hat{E} - \Delta & V \\ V^* & \hat{E} + \Delta \end{pmatrix}$$

in terms of these 4 matrices.

(b) Show that $\sigma_i \sigma_i = I$, and $\sigma_x \sigma_y = i \sigma_z$, $\sigma_y \sigma_z = i \sigma_x$, and $\sigma_z \sigma_x = i \sigma_y$.

(c) The spin of an electron can be written as

$$S = \frac{\hbar}{2} (\sigma_x \mathbf{i}_x + \sigma_y \mathbf{i}_y + \sigma_z \mathbf{i}_z)$$

If the Hamiltonian is given by $\mu \mathbf{B} \cdot \mathbf{S}$, where $B$ is the magnetic field, write out the Hamiltonian.

(d) The eigenvectors of the $\sigma_z$ are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Find the eigenvectors for $\sigma_x$ and $\sigma_y$. 