1.

We are told that the S-N curve for an elastic material follows the Basquin relationship, i.e.

\[ \sigma_a = C \cdot N_f^b \]

Where \( b \) is approximately equal to \(-0.09\). Say that the total lifetime of the component is \( n \) cycles. We are told that it spends 70\% of its life at the endurance limit \( \sigma_e \), 20\% at 1.1 \( \sigma_e \), and 10\% at 1.2 \( \sigma_e \). By definition, the lifetime at the endurance limit is \( N_f = 10^7 \) cycles so that:

\[ \sigma_e = C \left(10^7\right)^b \]

Say that the lifetime at \( \sigma_a = 1.1\sigma_e \) is \( N_1 \) cycles. \( N_1 \) is given by:

\[ 1.1\sigma_e = C \left(N_1\right)^b \]

Using the definition of \( \sigma_e \) we find that:

\[ N_1 = 10^7(1.1)^{(1/b)} = 3.468 \times 10^6 \]

Similarly, if the lifetime at \( \sigma_a = 1.2\sigma_e \) is \( N_2 \), \( N_2 \) is given by:

\[ N_2 = 10^7(1.2)^{(1/b)} = 1.319 \times 10^6 \]

We use the Palmgren-Miner law so that

\[ \frac{0.7n}{10^7} + \frac{0.2n}{3.468 \times 10^6} + \frac{0.1n}{1.319 \times 10^6} = 1 \]

Solving for \( n \), \( n = 4.912 \times 10^6 \). We can use the information given (failure occurs after 1/4 cycle) to determine the relationship between \( \sigma_e \) and \( \sigma_{TS} \) (not required for this problem but still interesting . . .)

\[ \sigma_{TS} = C \cdot (1/4)^b = 1.133C \]

\[ \sigma_e = C \cdot \left(10^7\right)^b = 0.266\sigma_{TS} \]

You should not necessarily assume that \( \sigma_e \approx 0.35\sigma_{TS} = 0.50\sigma_{TS} \). That only applies for some materials (some steels and copper alloys).
2.

Explain why the modified Goodman diagram can be re-written in terms of the endurance limit, as

\[ \sigma_e = \sigma_e|_{\sigma_m=0} \left(1 - \frac{\sigma_m}{\sigma_{TS}}\right) \]

where \( \sigma_e|_{\sigma_m=0} \) is the endurance limit for zero mean stress cyclic loading.

*Solution*

The modified Goodman equation states that:

\[ \sigma_a = \sigma_a|_{\sigma_m=0} \left(1 - \frac{\sigma_m}{\sigma_{TS}}\right) \]

What does this equation mean? Say we apply a certain stress with no mean stress (call that stress \( \sigma_a|_{\sigma_m=0} \)) and the component has a certain lifetime. Say we now have a situation with a mean stress \( \sigma_m \). This equation tells us the stress \( \sigma_a \) we can apply and have the lifetime be the same as in the case with no mean stress. This applies to any stress \( \sigma_a \), in particular we may let it be the endurance limit \( \sigma_e \) and then we obtain the desired relationship, i.e.

\[ \sigma_e = \sigma_e|_{\sigma_m=0} \left(1 - \frac{\sigma_m}{\sigma_{TS}}\right) \]

Since this a constant life relationship, the lifetime will be \( 10^7 \) cycles for both \( \sigma_e \) (with no mean stress applied) and \( \sigma_e|_{\sigma_m=0} \) (with mean stress applied), so both stresses (\( \sigma_e \) and \( \sigma_e|_{\sigma_m=0} \)) do indeed represent the endurance limits.
A circular cylindrical rod with a uniform cross-sectional area of 20 cm$^2$ is subjected to a mean axial force of 120 kN. The fatigue strength of the material, $\sigma_f = \sigma_{TF}$ is 250 MPa after $10^6$ cycles of fully reversed loading and $\sigma_{TS} = 500$ MPa. Using the different procedures discussed in class, estimate the allowable amplitude of force for which the shaft should be designed to withstand at least one million fatigue cycles. State all your assumptions clearly.

**Solution**

The different expressions we have to assess the influence of mean stresses are:

$$\sigma_a = \sigma_a |_{\sigma_m=0} \left\{ 1 - \frac{\sigma_m}{\sigma_y} \right\} \text{ (Soderberg)}$$

$$\sigma_a = \sigma_a |_{\sigma_m=0} \left\{ 1 - \frac{\sigma_m}{\sigma_{TS}} \right\} \text{ (Modified Goodman)}$$

$$\sigma_a = \sigma_a |_{\sigma_m=0} \left\{ 1 - \left( \frac{\sigma_m}{\sigma_{TS}} \right)^2 \right\} \text{ (Gerber)}$$

In all cases $\sigma_a |_{\sigma_m=0} = 250$ MPa and $\sigma_m = (120,000 \text{ N} / .0020 \text{ m}^2) = 60$ MPa. The Modified Goodman and Gerber criteria can be applied directly to give:

$$\sigma_a = 250 \left\{ 1 - \frac{60}{500} \right\} = 220 \text{ MPa, (Modified Goodman)}$$

$$\sigma_a = 250 \left\{ 1 - \left( \frac{60}{500} \right)^2 \right\} = 246.4 \text{ MPa, (Gerber)}$$

Applying the Soderberg criterion requires a bit more thought since it includes the yield strength (which is not given) rather than the tensile strength. Depending on the details of the material behavior, the yield stress $\sigma_{YS}$ could be the same as the tensile strength $\sigma_{TS}$ (for very brittle materials) or as low as $\approx 0.5\sigma_{TS}$ (for very ductile materials). I will assume that $\sigma_{YS} = 0.7\sigma_{TS} = 350$ MPa. Thus the Soderberg criterion gives:

$$\sigma_a = 250 \left\{ 1 - \frac{60}{350} \right\} = 207.1 \text{ MPa, (Soderberg)}$$

As you can see, you get significantly different answers depending on the model used. The *Soderberg* gives the most conservative result, while *Gerber* is the least conservative.
4.

\[ E = 210 \text{ GPa}, \quad A = 1000 \text{ MPa}, \quad E_f' = 1100 \text{ MPa}, \quad \gamma_f' = 2 \text{ GPa}, \quad b = -0.08 \]

\[ \gamma_f = 0.15, \quad c = -0.83 \]

SM07 PICKLING; RESIDUAL COMPRESSIVE STRESS, \( \sigma_{rc} = 250 \text{ MPa} \)

\[ (3.5) \quad \frac{\Delta E}{2} = \frac{\Delta E}{2E} + \left( \frac{\Delta E}{2A} \right)^{\gamma_f} \]

\[ (7.1) \quad \frac{\Delta E}{2} = 6_1 = 6_f \left( 2N_f \right)^b \]

\[ (3.5) \quad \frac{\Delta E}{2} = \frac{6_f'}{E} \left( 2N_f \right)^b + \frac{E_f'}{2A} \left( 2N_f \right)^c \quad \gamma_{m=0} = 0 \quad \text{AS FABRICATED} \]

\[
\begin{bmatrix}
\frac{\Delta E}{2} = -1100 \text{ MPa} \left( 2N_f \right)^{0.65} + 1.0 \left( 2N_f \right)^{0.65} \\
\end{bmatrix}
\]

\text{(FULLY REVERSED)}

For SM07 PEELED, \( \sigma_{m} = -250 \text{ MPa} \)

\[ \frac{\Delta E}{2} = \frac{6_f' - 6_m'}{E} \left( 2N_f \right)^b + \frac{E_f'}{2A} \left( 2N_f \right)^c \]

\[ \frac{\Delta E}{2} = 1350 \text{ MPa} \left( 2N_f \right)^{0.6} + 1.0 \left( 2N_f \right)^{0.6} \]

So... AS RECEIVED: \( \frac{\Delta E}{2} = \left( 5.74 \times 10^{-3} \right) \left( 2N_f \right)^{0.6} + \left( 2N_f \right)^{0.6} \)

SM07 PEELED: \( \frac{\Delta E}{2} = \left( 6.43 \times 10^{-3} \right) \left( 2N_f \right)^{0.6} + \left( 2N_f \right)^{0.6} \)

\text{SEE ATTACHED PLOT}
Strain Life, problem 8.1

- **as received**
- **shot peened**
Case 1:
For a non-closing notch, a fatigue crack will initiate in cyclic compression, due to the tensile residual stresses at the notch tip (see notes). The crack will propagate at a progressively decreasing rate, and will subsequently arrest because of crack closure, which becomes more of an issue at longer crack lengths.

Case 2:
There will be a larger zone of residual tensile stress initially. Therefore, the initial rate of crack growth is higher, and the crack will grow further than in case 1, before it arrests due to crack closure.

Case 3:
The first cycle could strain harden the material, resulting in a smaller plastic zone than that in Case 4. Assuming the large compressive overloads do not lead to early crack closure, the crack should grow longer than Case 1 and Case 2.
Case 4: It is generally the first cycle that dominates how the crack will grow. This is similar to case 3, but the crack should grow slightly longer due to the first cycle being more strongly compressive (which leads to a larger initial zone of residual tension).
Case I: Tensile overload results in an improvement in fatigue life.

As discussed in class, a tensile overload results in a region of residual compression ahead of a pre-existing crack. Subsequent tensile fatigue cycles must then overcome this residual stress before the crack tip experiences a tensile load. This will improve the fatigue life.

Case II: Tensile overload results in a reduction in fatigue life.

It is possible for the tensile overload to initiate new cracks that would not have ordinarily been present during less severe cycling. These new cracks can propagate under the subsequent loading and lead to premature failure. This is why structural components are not intentionally subjected to overloads prior to going into service.

Case III: Tensile overload leads to no change in fatigue life.

An initially defect-free, smooth surface specimen of relatively high strength might be able to sustain an overload without having any cracks nucleate. Assuming the strength is sufficiently high such that the overload did not harden the material, the fatigue life would be roughly the same.
7.

(14.2) Plot the load vs. crack opening displacement curve for a metallic material subjected to loading and unloading phases in zero-tension zero-fatigue.

(a) Plastic deformation at the crack tip: no change in crack configuration (i.e., crack opening and crack length), loading phase.

Because there is no change in crack configuration, \( \frac{dP}{dS} \rightarrow \text{infinite initially.}\) Plastic deformation causes an increase in \( S \).

(b) Gradual opening of the crack during the loading phase and plastic deformation at the tip:

\[
\sigma = \frac{E}{1-v} \frac{P_0}{a} \quad \text{Eqn 9.23}
\]

\[
S = \frac{P_0 a}{E} \left( 1 - \frac{v}{2} \right)
\]

As load increases, \( P \), compliance increases and stiffness decreases as crack advances.

(c) Elastic behavior at constant crack configuration during the loading phase.

\[
P \uparrow \Rightarrow \text{elastic deformation} \Rightarrow S
\]

(d) Effect of plastic behavior on crack opening and configuration change on the \( P \& S \) plot during the unloading phase.

\[
S(0) = \text{unloading} = S_0
\]

\( S_0 \) = difference in \( S \) values between the "sawcut" and fatigue crack.
(c) Effect of plastic behavior > effect of configuration change on the P-s plot during the unloading phase:

Unloading slope is lower than in (a) because of plasticity increases compliance.

(b) Effect of plastic behavior < effect of configuration change:

Unloading slope is higher than in (a) closer to (c).

(a) Configuration change during the unloading with negligible plastic deformation at the crack tip:

Because of crack, compliance ↑, slope decreases.