18.01 Single Variable Calculus
Fall 2006

# 18.01 Practice Questions for Exam 3 – Fall 2006

1. Evaluate  a) $\int_0^1 \frac{x\,dx}{\sqrt{1+3x^2}}$  b) $\int_{\pi/3}^{\pi/2} \cos^3 x \, \sin 2x \, dx$

2. Evaluate $\int_0^1 x\,dx$ directly from its definition as the limit of a sum.

   Use upper sums (circumscribed rectangles). You can use the formula $\sum_1^n i = \frac{1}{2}n(n+1)$.

3. A bank gives interest at the rate $r$, compounded continuously, so that an amount $A_0$ deposited grows after $t$ years to an amount $A(t) = A_0 e^{rt}$.

   You make a daily deposit at the constant annual rate $k$; in other words, over the time period $\Delta t$ you deposit $k\Delta t$ dollars. Set up a definite integral (give reasoning) which tells how much is in your account at the end of one year. (Do not evaluate the integral.)

4. Consider the function defined by $F(x) = \int_0^x \sqrt{3 + \sin t}\, dt$. Without attempting to find an explicit formula for $F(x)$,

   a) (5) show that $F(1) \le 2$;

   b) (5) determine whether $F(x)$ is convex ("concave up") or concave ("concave down") on the interval $0 < x < 1$; show work or give reasoning;

   c) (10) give in terms of values of $F(x)$ the value of $\int_1^2 \sqrt{3 + \sin 2t}\, dt$.

5. If $\int_0^x f(t)\,dt = e^{2x}\cos x + c$, find the value of the constant $c$ and the function $f(t)$.

6. A glass vase has the shape of the solid obtained by rotating about the $y$-axis the area in the first quadrant lying over the $x$-interval $[0, a]$ and under the graph of $y = \sqrt{x}$. By slicing it horizontally, determine how much glass it contains.

7. A right circular cone has height 5 and base radius 1; it is over-filled with ice cream, in the usual way. Place the cone so its vertex is at the origin, and its axis lies along the positive $y$-axis, and take the cross-section containing the $x$-axis. The top of this cross-section is a piece of the parabola $y = 6 - x^2$. (The whole filled ice-cream cone is gotten by rotating this cross-section about the $y$-axis.)

   What is the volume of the ice cream? (Suggestion: use cylindrical shells.)

8. Rectangles are inscribed as shown in the quarter-circle of radius $a$, with the point $x$ being chosen randomly on the interval $[0, a]$. Find the average value of their area.

9. Find the approximate value given for the integral below by the trapezoidal rule and also by Simpson's rule, taking $n = 2$ (i.e., dividing the interval of integration into two equal subintervals):

$$\int_0^{\pi/2} \sin^6 x \, dx$$

Other possible problems: Volumes by vertical slicing 4B, Work problems (P.Set 5)