1. Discussed the approach to the Green’s Kernel via the Laplace operator. Let \( p(D) \) be a constant coeff. linear differential operator of order \( n+1 \) and let \( f(t) \) be a function of exponential type. Let \( y(t), \ t \geq 0 \) be the solution of the IVP

\[
\left\{ \begin{array}{ll}
p(D)y = f(t) & \\
y(0) = y_0 \\
\vdots \\
y^{(n)}(0) = y_0^{(n)}
\end{array} \right.
\]

Let \( y_n(t) \) be the sol’n of

\[
\left\{ \begin{array}{ll}
p(D)y = 0 & \\
y(0) = y_0 \\
\vdots \\
y^{(n)}(0) = y_0^{(n)}
\end{array} \right.
\]

let \( k(t) \) be the sol’n of

\[
\left\{ \begin{array}{ll}
p(D)y = 0 & \\
y(0) = 0 \\
y^{(n)}(0) = 0
\end{array} \right.
\]

Then we have \( p(s) \mathcal{L}[y] - Q(s) = F(s) \)

Then we have \( p(s) \mathcal{L}[y] - Q(s) = F(s) \)

for some poly. \( Q(s) \) of degree \( \leq n \) and

\[
F(s) = \mathcal{L}[f(t)]. \quad \mathcal{L}[y] = \frac{Q(s)}{p(s)} + \frac{1}{p(s)}F(s). \quad \text{Moreover, we also have}
\]

\[
\mathcal{L}[y_n] = \frac{Q(s)}{p(s)} \quad \text{and} \quad \mathcal{L}[k] = \frac{1}{p(s)}.
\]

Therefore,

\[
y(t) = y_n(t) + \int_0^t k(t-u)f(u)du.
\]

Moreover \( y_p(t) = \int_0^t k(t-u)f(u)du \). Therefore,

\[
y(t) = y_n(t) + \int_0^t k(t-u)f(u)du.
\]

2. Worked the IVP \( y'' + 2y' + y = f(t) = te^{2t} \) by Green’s Kernel method

\[
y(0) = 1 \\
y'(0) = 0
\]

And saw it involved more computation than the usual Laplace operator method (which we did in lecture on Monday).

3. Very quickly reviewed what a system of linear ODE’s is, introduced matrix notation for such a system,

\[
y' = Ay + F(t)
\]

and argued that a formal solution should be of the form

\[
y = \exp[tA] \cdot y_0 + \exp[tA] \cdot \int_0^t \exp[-uA] F(u)du,
\]

once we make sense of all this.