1. Study the phase portraits of the systems
\[
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & \epsilon \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]
and
\[
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ \epsilon & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
\]

2. Consider
\[
x' = y - x(x^2 + y^2), \quad y' = -x - y(x^2 + y^2).
\]
(a) Find the critical point.
(b) Determine the stability of the linear approximation at \((0, 0)\).
(c) Determine the stability of \((0, 0)\).
(d) Repeat for
\[
x' = y + x(x^2 + y^2), \quad y' = -x - y(x^2 + y^2).
\]

3. In the competitive system
\[
x' = x(k - ax - by), \quad y' = y(m - cx - dy), \quad k, m, a, b, c, d > 0
\]
if the lines \(ax + by = k\) and \(cx + dy = m\) do not intersect in the first quadrant \(x, y > 0\) find the limit set.

4. If \((x(t), y(t))\) is a solution of the predator-prey equations
\[
x' = x(-k + by), \quad y' = y(m - cx), \quad k, m, b, c > 0
\]
of period \(T > 0\), show that
\[
\frac{1}{T} \int_0^T x(t)dt = \frac{m}{c}, \quad \frac{1}{T} \int_0^T y(t)dt = \frac{k}{b}.
\]

5. (a) Show that the differential equation
\[
x'' + (x^2 + 2(x')^2 - 1)x' + x = 0
\]
has a nontrivial periodic solution.
(b) Show that the system of differential equations
\[
x' = x + y^2 + x^3, \quad y' = -x + y + xy^2
\]
has no nontrivial periodic solution.