Commodity Modeling

• The views represented herein are the author’s own views and do not necessarily represent the views of Morgan Stanley or its affiliates, and are not a product of Morgan Stanley Research.
Trader benefits from low prices

First reported 03/11/2009

Dow Jones & Company Inc

Trafigura: May Have Best Earnings Ever In Fiscal 2009

• SINGAPORE -(Dow Jones)- International commodities trading firm Trafigura Beheer B.V. is potentially on track to post its best results ever in fiscal 2009 on lower oil prices and contango markets, a company executive said Wednesday.
WTI futures contracts: Jan. 15, 2009

Source: Bloomberg
Trading in Contango Markets

\[ F_{\text{Feb'09}} = 35 \$/\text{bbl} \quad F_{\text{Feb'10}} = 60 \$/\text{bbl} \]

Strategy: On Jan. 15, 2009
- Borrow $35 \Rightarrow Buy 1 bbl \Rightarrow Store
- Short Feb’10 futures contract (1 bbl)
- Lock-in profit: $25 - Interest Payment
  - \text{Interest Payment} = 35*r
  - If \ r = 10\% 
    \[ \text{Interest Payment} = 3.5/\text{bbl} \quad \text{Profit} = 21.5/\text{bbl} \]
Summary: to generate profit

• Needed asset (storage)
• Needed strategy:
  – Long Feb’09 contract
  – Short Feb’10 contract
  – Or long Feb-Feb calendar spread
What if you need to lease storage from Aug to Dec
How much will you pay for this lease on Jan 1?

• $F_{Aug} = 55 \$/bbl \hspace{1cm} F_{Dec} = 58 \$/bbl$

Source: Bloomberg

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
This is what the trader will do

On Jan 1

• Buy Aug/Dec spread:
  – Long Aug futures contract
  – Short Dec futures contract

• On Aug 1
  – buy 1 bbl for $55/bbl and store it

• Wait till Dec and then sell 1 bbl for $58

• Lock-in $3/bbl. Can pay for storage up to $3/bbl

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
This is what the quant will do

• On Jan 1 sell Aug/Dec spread option:

\[ \text{Payout at exercise} = \left[ F_{Dec} - F_{Aug} \right]^+ \]

• Exercise date Jul 31
• Interest rates are ignored for simplicity (should not be)
Why is this better?

- The value of this calendar spread option

\[
V = \left( F_{Dec} N\left(d_1\right) - F_{Aug} N\left(d_2\right) \right) \cdot DiscFactor
\]

\[
d_1 = \left[ \log \frac{F_{Dec}}{F_{Aug}} + \frac{\sigma_T^2}{2} T \right] \sqrt{\sigma_T^2 T} \\
n_2 = d_1 - \sigma_T \sqrt{T}
\]

\[
\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho \sigma_1 \sigma_2}
\]

\[- V = 4.4677 \$/bb\]

The value is always greater than the spread because the spread is its intrinsic value.

Developed for educational use at MIT and for publication through MIT OpenCourseware.

No investment decisions should be made in reliance on this material.
The benefit:

• Storage bid can be increased to $4.46/bbl increasing the likelihood of winning the deal. We can also keep a greater profit.

• Is there the risk? What if on Jul 31
  \[ F_{\text{Aug}} = 65 \text{ $/bbl} \quad F_{\text{Dec}} = 80 \text{ $/bbl} \]
  and we owe $15/bbl to the option holder

• No worry: We have storage ➔ On Jul 31
  Buy Aug crude for 65 $/bbl and simultaneously
  Sell Dec crude for 80 $/bbl using Dec futures contract
  Lock-in $15 $/bbl to repay option holder
In reality …

• Sell portfolio of spread option
• Satisfy a number of physical constraints
  – Injection rates
  – Withdrawal rates
  – Do not inject more than max capacity
  – Do not withdraw from the empty tank
  – etc
Storage optimization

• Find

\[ V = \max \left\{ \sum_{i<j} x_{i,j} S_{i,j} + \sum_{i<j} \nu_{i,j} U_{i,j} - \sum_i y_i F_i + \sum_j z_j F_j \right\} \]

\[ x \geq 0, \nu \geq 0, y \geq 0, z \geq 0 \]

• \( F_i, F_j \) - today’s futures prices for contracts expiring at times \( T_i \) and \( T_j \)

• \( y_i, z_j \) - volumes committed today for injection at time \( T_i \), or withdrawal at \( T_j \)

Developed for educational use at MIT and for publication through MIT OpenCourseware.

No investment decisions should be made in reliance on this material.
Storage Optimization

• $S_{i,j}$ - is the value of the option to inject at time $i$ and withdraw at time $j$

\[
Payout\_at\_exercise = \max \left\{ F_j - F_i - Cost, \ 0 \right\}
\]

• $U_{i,j}$ - is the value of the option to withdraw at time $i$ and inject at later time $j$

\[
Payout\_at\_exercise = \max \left\{ F_i - F_j - Cost, \ 0 \right\}
\]

• $x_{i,j}, v_{i,j}$ - option volumes sold against the storage today

Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.
Constraints

• Let’s introduce Boolean “in-the-money at exercise” variables

\[
\Omega_{i,j}^S = \begin{cases} 
1 & \text{if option } S_{i,j} \text{ expires in-the-money} \\
0 & \text{otherwise}
\end{cases}
\]

\[
\Omega_{i,j}^U = \begin{cases} 
1 & \text{if option } U_{i,j} \text{ expires in-the-money} \\
0 & \text{otherwise}
\end{cases}
\]
Constraints

• Injection constraints

\[ \sum_{i<j} x_{i,j} \Omega_{i,j}^S - \sum_{\ell<i} x_{\ell,i} \Omega_{i,j}^S + \sum_{\ell<i} v_{\ell,i} \Omega_{i,j}^U - \sum_{i<j} v_{i,j} \Omega_{i,j}^U + y_i - z_i \leq I_i \quad i = 1, \ldots, N \]

• Withdrawal constraints

\[ \sum_{i<j} x_{i,j} \Omega_{i,j}^S - \sum_{\ell<i} x_{\ell,i} \Omega_{i,j}^S + \sum_{\ell<i} v_{\ell,i} \Omega_{i,j}^U - \sum_{i<j} v_{i,j} \Omega_{i,j}^U + y_i - z_i \geq -W_i \quad i = 1, \ldots, N \]
Constraints

- **Maximum capacity constraints**

\[ C_0 + \sum_{k \leq i} \left\{ \sum_{j > i} x_{k,j} \Omega_{k,j}^S - \sum_{j > i} v_{k,j} \Omega_{k,j}^U + y_k - z_k \right\} \leq C_i^{\text{max}} \quad i = 1, \ldots, N \]

- **Minimum capacity constraints**

\[ C_0 + \sum_{k \leq i} \left\{ \sum_{j > i} x_{k,j} \Omega_{k,j}^S - \sum_{j > i} v_{k,j} \Omega_{k,j}^U + y_k - z_k \right\} \geq C_i^{\text{min}} \quad i = 1, \ldots, N \]
Solution

• Approximation
• Monte-Carlo simulation
• Alternative approach: Stochastic control

Additional complications

• There is no spread option market now: we cannot sell spread option directly
• We must design a strategy of replicating selling the spread option
• Similar to Black-Scholes delta-hedging strategy
Power Plant

• Spark Spread Option

• Merchant Power Plant
  – Should be run if the market price of power is higher than the cost of fuel plus variable operating costs

Net Profit from this operating strategy is:

$$\Pi = \max\left( Price_{power} - \frac{Heat - Rate}{1000} Price_{fuel} - Variable - Costs, 0 \right)$$

Operating a merchant power plant is financially equivalent to owning a portfolio of daily options on spreads between electricity and fuel (spark spread options)
Properties of energy prices

• Behavior of energy prices is unique

Example 1: Fat Tails of distributions

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.
Properties of energy prices

Example 1: Fat Tails of distributions

Source: Eydeland, Wolyniec
Properties of energy prices


<table>
<thead>
<tr>
<th></th>
<th>Annual Volatility</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nord Pool</td>
<td>182%</td>
<td>1.468</td>
<td>26.34</td>
</tr>
<tr>
<td>NP 6.p.m.</td>
<td>238%</td>
<td>2.079</td>
<td>76.82</td>
</tr>
<tr>
<td>DAX</td>
<td>23%</td>
<td>0.004</td>
<td>3.33</td>
</tr>
</tbody>
</table>

Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.
Special properties of electricity prices: spikes, high volatility

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
Special properties of electricity prices

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.

No investment decisions should be made in reliance on this material.
Special properties of electricity prices

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.
Behavior of power prices

- Mean reversion
- spikes
- high kurtosis
- regime switching
- lack of data
- non-stationarity
Correlation between power and gas also has unique structure. If the model does not capture this structure, it may misprice spread options (tolling contracts, power plants, etc.)

Source: Eydeland, Woyniec
Models

• Spot Processes
• GBM

\[ dS_t = \mu S_t \, dt + \sigma S_t \, dW_t \]

• GBM with mean reversion

\[ \frac{dS_t}{S_t} = \kappa (\theta - \log S_t) \, dt + \sigma \, dW_t \]

• + jumps

\[ \frac{dS_t}{S_t} = (\mu - \lambda k) \, dt + \sigma \, dW_t + (Y_t - 1) \, dq_t \]

• + jumps and mean reversion

\[ \frac{dS_t}{S_t} = (\mu - \lambda k - \log S_t) \, dt + \sigma \, dW_t + (Y_t - 1) \, dq_t \]
More complicated models

• Models with stochastic convenience yield
• Models with stochastic volatility
• Regime switching models
• Models with multiple jump processes
• Various term structure models
Spot Processes: Cons

Difficult to use for power products due to non-storability:

• No no-arbitrage argument
• How to price forward contracts and options?
  – In the case of storable commodities (NG, CL) we need convenience yield.
  – Calibration is difficult to implement due to overlapping data.
  – Cannot model the correlation structure between forward contracts.
  – Cannot model complex volatility structures.
  – Spot processes without jumps or stochastic volatility generate unrealistic power price distributions.
  – Cannot capture complex power/gas correlation structure.
A different approach
Hybrid Model: Stack Method

Price formation mechanism: Bid stack

| Generator 1. | Price ($/MWh) | 20  | 25  | 30  | 35  | 50  |
|             | Volume (MWh)  | 50  | 100 | 200 | 400 | 600 |
| Generator 2. | Price ($/MWh) | 18  | 40  | 100 |
|             | Volume (MWh)  | 100 | 200 | 500 |

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.

No investment decisions should be made in reliance on this material.
Hybrid Models: Stack Method

Bid stack:

<table>
<thead>
<tr>
<th>Price ($/MWh)</th>
<th>18</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (MWh)</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>300</td>
<td>500</td>
<td>600</td>
<td>800</td>
<td>1100</td>
</tr>
</tbody>
</table>

Source: Eydeland, Wolyniec

\[ P_t = S^{\text{bid}}(D_t) \]

Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.
Drivers:

1. Demand

2. Fuel Prices

3. Outages

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
How to build the bid stack?

1. Fuel + Outages → Generation Stack

2. Generation stack → Bid Stack

Transformation at step 2 matches market data and preserves higher moments of price distribution (skewness, kurtosis)
Fuel Model

Group 1
- Natural Gas
- #2 Heating Oil
- #6 Fuel Oil (with different sulfur concentration)
- Coal
- Jet Fuel
- Diesel
- Methane
- Liquefied Natural Gas (LNG)
- Etc.

Group 2
- Nuclear
- Hydro
- Solar
- Wind
- Biomass
- Etc.

Prices of Group 1 fuels are modeled using term structure models, matching forward prices, option prices and correlation structure.
Outage Model

Standard process (e.g., Poisson) utilizing EFOR (Equivalent Forced Outage Rate)

As a result for each time $T$ we have an outage vector

$$\Omega_T=(\omega_{T,1}, \ldots, \omega_{T,L})$$

$\omega_{T,i}=1$ if at time $T$ the unit $I$ is experiencing forced outages

$\omega_{T,i}=0$ otherwise
Demand

Demand can be modeled as a function of temperature

\[ D_t = d(t, \mathcal{Z}_t) \]

Temperature evolution process:

i. evolution of the principal modes
ii. evolution of the daily perturbations
Power Prices

\[ P_T = s_T^{\text{bid}} (D_T) \equiv \alpha_1 s_T^{\text{gen}} (D_T; T, U_T, \Omega_T (\alpha_2 \lambda), E_T, VOM_T, \alpha_3 C_T) \]

The constants \( \alpha \) chosen to match market data
Justification

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.
Justification: PJM Prices - actual vs. model

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
## Justification

Skewness and kurtosis of PJM price distribution: model vs. empirical data

<table>
<thead>
<tr>
<th></th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model data</td>
<td>Empirical data</td>
</tr>
<tr>
<td>Summer 2000</td>
<td>3.58</td>
<td>3.17</td>
</tr>
<tr>
<td>Summer 2001</td>
<td>18.13</td>
<td>14.65</td>
</tr>
<tr>
<td>Winter 2000</td>
<td>.68</td>
<td>1.32</td>
</tr>
<tr>
<td>Winter 2001</td>
<td>.18</td>
<td>1.54</td>
</tr>
</tbody>
</table>

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
Simulated correlation structure

Source: Eydeland, Wolyniec

Developed for educational use at MIT and for publication through MIT OpenCourseware.
No investment decisions should be made in reliance on this material.
vs actual correlation structure

Source: Eydeland, Wolyniec
References


Disclosures

The information herein has been prepared solely for informational purposes and is not an offer to buy or sell or a solicitation of an offer to buy or sell any security or instrument or to participate in any trading strategy. Any such offer would be made only after a prospective participant had completed its own independent investigation of the securities, instruments or transactions and received all information it required to make its own investment decision, including, where applicable, a review of any offering circular or memorandum describing such security or instrument, which would contain material information not contained herein and to which prospective participants are referred. No representation or warranty can be given with respect to the accuracy or completeness of the information herein, or that any future offer of securities, instruments or transactions will conform to the terms hereof. Morgan Stanley and its affiliates disclaim any and all liability relating to this information. Morgan Stanley, its affiliates and others associated with it may have positions in, and may effect transactions in, securities and instruments of issuers mentioned herein and may also perform or seek to perform investment banking services for the issuers of such securities and instruments.

The information herein may contain general, summary discussions of certain tax, regulatory, accounting and/or legal issues relevant to the proposed transaction. Any such discussion is necessarily generic and may not be applicable to, or complete for, any particular recipient's specific facts and circumstances. Morgan Stanley is not offering and does not purport to offer tax, regulatory, accounting or legal advice and this information should not be relied upon as such. Prior to entering into any proposed transaction, recipients should determine, in consultation with their own legal, tax, regulatory and accounting advisors, the economic risks and merits, as well as the legal, tax, regulatory and accounting characteristics and consequences, of the transaction.

Notwithstanding any other express or implied agreement, arrangement, or understanding to the contrary, Morgan Stanley and each recipient hereof are deemed to agree that both Morgan Stanley and such recipient (and their respective employees, representatives, and other agents) may disclose to any and all persons, without limitation of any kind, the U.S. federal income tax treatment of the securities, instruments or transactions described herein and any fact relating to the structure of the securities, instruments or transactions that may be relevant to understanding such tax treatment, and all materials of any kind (including opinions or other tax analyses) that are provided to such person relating to such tax treatment and tax structure, except to the extent confidentiality is reasonably necessary to comply with securities laws (including, where applicable, confidentiality regarding the identity of an issuer of securities or its affiliates, agents and advisors).

The projections or other estimates in these materials (if any), including estimates of returns or performance, are forward-looking statements based upon certain assumptions and are preliminary in nature. Any assumptions used in any such projection or estimate that were provided by a recipient are noted herein. Actual results are difficult to predict and may depend upon events outside the issuer’s or Morgan Stanley’s control. Actual events may differ from those assumed and changes to any assumptions may have a material impact on any projections or estimates. Other events not taken into account may occur and may significantly affect the analysis. Certain assumptions may have been made for modeling purposes only to simplify the presentation and/or calculation of any projections or estimates, and Morgan Stanley does not represent that any such assumptions will reflect actual future events. Accordingly, there can be no assurance that estimated returns or projections will be realized or that actual returns or performance results will not be materially different than those estimated herein. Any such estimated returns and projections should be viewed as hypothetical. Recipients should conduct their own analysis, using such assumptions as they deem appropriate, and should fully consider other available information in making a decision regarding these securities, instruments or transactions. Past performance is not necessarily indicative of future results. Price and availability are subject to change without notice.

The offer or sale of securities, instruments or transactions may be restricted by law. Additionally, transfers of any such securities, instruments or transactions may be limited by law or the terms thereof. Unless specifically noted herein, neither Morgan Stanley nor any issuer of securities or instruments has taken or will take any action in any jurisdiction that would permit a public offering of securities or instruments, or possession or distribution of any offering material in relation thereto, in any country or jurisdiction where action for such purpose is required. Recipients are required to inform themselves of and comply with any legal or contractual restrictions on their purchase, holding, sale, exercise of rights or performance of obligations under any transaction. Morgan Stanley does not undertake or have any responsibility to notify you of any changes to the attached information.

With respect to any recipient in the U.K., the information herein has been issued by Morgan Stanley & Co. International Limited, regulated by the U.K. Financial Services Authority. THIS COMMUNICATION IS DIRECTED IN THE UK TO THOSE PERSONS WHO ARE MARKET COUNTERPARTIES OR INTERMEDIATE CUSTOMERS (AS DEFINED IN THE UK FINANCIAL SERVICES AUTHORITY’S RULES).

ADDITIONAL INFORMATION IS AVAILABLE UPON REQUEST. Developed for educational use at MIT and for publication through MIT OpenCourseware. No investment decisions should be made in reliance on this material.