Solution of Problem Set 8

Assigned: April 11, 2008
Due: April 18, 2008

Problem 1: Nise Problem 7-1 (p. 357 5th Ed.).

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Problem 2: Nise Problem 7-15 (p. 358 5th Ed.).

Collapsing the inner loop and multiplying it by \( \frac{1000}{s} \) yields the following equivalent forward-path transfer function, corresponding to a type 1 system:

\[
G_e(s) = \frac{10^5(s + 2)}{s(s^2 + 1005s + 2000)}
\]

MATLAB Command – line:

```matlab
>> G1=feedback(tf(100*[1 2],[1 5 0]),10)
```

Transfer function:

\[
100 \ s + 200
\]

\[
----------------------
\]

\[
s^2 + 1005 \ s + 2000
\]

```matlab
>> G2=series(G1,tf(1000,[1 0]))
```

Transfer function:

\[
100000 \ s + 200000
\]

\[
----------------------
\]

\[
s^3 + 1005 \ s^2 + 2000 \ s
\]
Problem 3: Nise Problem 7-59 (p. 365 5th Ed.).

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Problem 4: Nise Problem 7-60 (p. 365 5th Ed.).

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Problem 5: Nise Problem 6-33 (p. 314 5th Ed.)

\[
\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{K(s + 4)}{s^3 + 3s^2 + (K + 2)s + 4K}
\]

As a necessary condition for the stability, all the coefficients of the denominator should have the same sign which requires \((K + 2) > 0, (4K) > 0 \Rightarrow K > 0 \). Furthermore, we compute
marginal value of K by comparing the denominator with standard format of \((s+a)(s^2+\omega^2) = s^3 + as^2 + \omega^2 s + a\omega^2\). By matching those two third order polynomials we realize that:

\[
\begin{align*}
\frac{a}{\omega^2} &= 4K \
\omega^2 &= (K+2) \
\omega^2 &= 4K 
\end{align*}
\]

Hence the range of \(0 < K < 6\) keeps the system stable. The system has an undamped oscillation for \(K = 6\), corresponding to \(\omega = 2\sqrt{2} \text{ rad/sec}\).

**Problem 6:** Nise Problem 6-26 (p. 314 5th Ed.).

\[
G(s) = \frac{K(s-2)(s+2)}{s^2+3} = \frac{K(s^2-4)}{s^2+3}
\]

\[
\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{K(s^2-4)}{(K+1)s^2+(3-4K)}
\]

As a necessary condition for the stability, all the coefficients of the denominator should have the same sign which requires \((K+1)(3-4K) > 0 \Rightarrow -1 < K > \frac{3}{4}\). However, for that range of \(K\) the system has only two purely oscillatory poles without any damping. Hence, the system is not truly stable, but only marginally stable.