Lecture 6
Moderately Large Deflection Theory of Beams

Problem 6-1:
Part A: The department of Highways and Public Works of the state of California is in the process of improving the design of bridge overpasses to meet earthquake safety criteria. As a highly paid consultant to the project, you were asked to evaluate its soundness. You rush back to your lecture notes, and you model the overpass as a simply supported beam of span $L$ with an overhang $\Delta = 0.01L$. Assume that the distributed load is a sinusoidal function.

![Beam Diagram]

a) Calculate the maximum allowable midspan deflection $(w_o)_{critical}$ under which the beam will slide off its support.

Part B: Assume that the above design with an external axial force $N=0$ and $\Delta=0.01L$ has a safety factor of one. The design of earthquake resistant structures requires a safety factor of five, meaning that $(w_o)_{critical}$ must be increased by a factor of five without the bridge collapsing. Two possible design modifications were proposed. In the first one, the overhang is simply increased to $\Delta_{new}$. In the second design, a tensile force $N$ is applied to the bridge to increase its transverse stiffness and thus reduce the central deflection and the resulting motion of the support.

b) For the first proposed modification, what length $\Delta_{new}$ of the overhang will meet the requirement of a safety factor of five? Give your result in terms of the original $\Delta$ and other parameters if needed.

c) For the second design, what is the magnitude of the dimensionless tensile force $N/EA$ that will give a safety factor equal to five?

d) Which design is better? Can you think of a third alternative design solution?
**Problem 6-2:**
A long span aerial tramway steel cable of length \( L = 1 \text{km} \) is loaded by a hurricane wind with intensity \( q(x) \) sinusoidally distributed between the end stations. The cable deflects by \( w_0 = 5 \text{m} \).

\[
E = 2.1 \times 10^5 \text{ MPa} \\
\sigma_y = 300 \text{ MPa} \\
D = 60 \text{ mm}
\]

\[
q(x) = q_0 \sin\left(\frac{\pi x}{L}\right)
\]

\( \text{Cross-section of cable} \)

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a) Calculate the resulting load intensity \( q_0 \)

b) Calculate the tension in the cable \( N \).

c) Calculate the tensile stress.

d) Compare (c) with the yield stress, and determine the safety factor.

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**Problem 6-3:**
Plot the dimensionless deflections \( \left(\frac{w_0}{L}\right) \) versus the dimensionless line load for both bending and membrane (cable) solutions over a slender beam. At what dimensionless deflections will the bending and membrane solutions be equal, assuming a length to thickness ratio equal to 10?