Notes for Lecture 3

Example of Independence

$A = \{i = 2 \text{ or } 3\}$;  
$B = \{j = 1 \text{ or } 5 \text{ or } 6\}$.

Thus, we have

$A \cap B = \{(2,1), (3,1), (2,5), (3,5), (2,6), (3,6)\}$.

So, we can compute the following:

$P(A) = 12/36 = 1/3$;  
$P(B) = 18/36 = 1/2$;  
$P(A \cap B) = 6/36 = 1/6 = P(A)P(B)$.

We can also demonstrate the independence in the following way.

Let

$\text{prob}(ij) = f(i)g(j)$

Thus,
\[ \text{prob}(A) = f(2)g(1) + f(2)g(2) + \cdots + f(2)g(6) \\
+ f(3)g(1) + f(3)g(2) + \cdots + f(3)g(6) \\
= (f(2) + f(3))(g(1) + g(2) + \cdots + g(6)) \]

Similarly, we have
\[ \text{prob}(B) = (g(1) + g(5) + g(6))(f(1) + f(2) + \cdots + f(6)) \]

And
\[ \text{prob}(A \cap B) = f(2)g(1) + f(3)g(1) + f(2)g(5) + f(3)g(5) + f(2)g(6) + f(3)g(6) \]

Note that
\[ f(1) + f(2) + \cdots + f(6) = 1 \]

And
\[ g(1) + g(2) + \cdots + g(6) = 1 \]

Therefore,
\[ \text{prob}(A) = f(2) + f(3) \]

\[ \text{prob}(B) = g(1) + g(5) + g(6) \]

Thus,
\[ \text{prob}(A) \text{prob}(B) = (f(2) + f(3))(g(1) + g(5) + g(6)) \]
\[ = f(2)g(1) + f(3)g(1) + f(2)g(5) + f(3)g(5) + f(2)g(6) + f(3)g(6) \]
\[ = \text{prob}(A \cap B) \]

So, A and B are independent.