Chapter 10. Meeting 10, Approaches: Probability and Markov Chains

10.1. Announcements

• Musical Design Report 2 due this Thursday, 11 March
• Thursday we will work in PD and Csound
• Quiz next Tuesday

10.2. Half-Period Oscillators as ParameterObjects

• Continuously varying the seconds per cycle (frequency) of an oscillator results in complex periodicities; random or discrete frequency variation results in complexity

:: tpmap 100 ws,e, (ls,e, 50, 10, 30), 0, 0, 10
waveSine, event, (lineSegment, (constant, 50), (constant, 10), (constant, 30)), 0, (constant, 0), (constant, 10)
TPmap display complete.

:: tpmap 100 ws,e, (ru, 19, 21), 0, 0, 10
waveSine, event, (randomUniform, (constant, 19), (constant, 21)), 0, (constant, 0), (constant, 10)
TPmap display complete.

• An alternative is an oscillator that only updates seconds per half cycle (half frequency) once per half-period
WaveHalfPeriodSine, WaveHalfPeriodTriangle, WaveHalfPeriodPulse, WaveHalfPeriodCosine

:: tpmap 100 whps,e,(bg,rp,(2,6,10,14,18)),0,0,10
waveHalfPeriodSine, event, (basketGen, randomPermutate, (2,6,10,14,18)), 0,
(constant, 0), (constant, 10)
TPmap display complete.

:: tpmap 100 whpt,e,(bg,rp,(2,6,10,14,18)),0,0,10
waveHalfPeriodTriangle, event, (basketGen, randomPermutate, (2,6,10,14,18)), 0,
(constant, 0), (constant, 10)
TPmap display complete.

:: tpmap 100 whpp,e,(bg,rp,(2,6,10,14,18)),0,0,10
waveHalfPeriodPulse, event, (basketGen, randomPermutate, (2,6,10,14,18)), 0,
(constant, 0), (constant, 10)
TPmap display complete.

10.3. Markov Analysis and Generation: Basics

- Examine an ordered sequence states
- Given an event at \( n-1 \), what is the probability of any state (of all possible states) at \( n \)?
• Look at all possible \( n-1 \) states, and find how often they move to each state at \( n \)

• Use these probabilities to re-generate new sequences (where more frequent states result in proportionally weighted randomness)

10.4. Markov Analysis and Generation: Orders

• Zeroth order: examine 0 past states; given all possible states, generate \( n \) based on the distribution of all states.

• First order: examine 1 past state; generate \( n \) based on the probability of \( n-1 \) moving to each state.

• Second order: examine 2 past states; generate \( n \) based on the probability of \( n-2 \) and \( n-1 \) moving to each state.

• Second order: examine 3 past states; generate \( n \) based on the probability of \( n-3 \), \( n-2 \) and \( n-1 \) moving to each state.

• The greater the order, the more the past is taken into account in determining the next state

• The greater the order, the more the output is similar to the source


• What does Ames refer to by stationary probabilities

• What does Ames claim as the greatest strength of Markov chains?

• What technique does Ames suggests as a way to create large-scale behavior out of Markov chains?

10.6. Markov Chains: History

• 1906: Andrey Andreyevich Markov, Russian mathematician

  Used Markov chains to show tendencies in written Russian in a text by Pushkin

• 1949: Claude E. Shannon and Warren Weaver: A Mathematical Theory of Communication; associated with information theory

  • Demonstrate using stochastic processes to generate English sentences

  • Suggest application to any sequence of symbols, including music
10.7. Markov Chains: History: Early Musical Applications

• The “Banal Tune-Maker” of Richard C. Pinkerton (1956)

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**Transition Probabilities** show how frequently any note follows any other in the 39 nursery tunes. The first notes of all possible pairs are listed in the column at the left; the second notes, in the row at the top. Thus each number in the table gives the probability that the note at the top of its column will come after the note at the left of its row. The color pattern divides the table between likely transitions (colored) and unlikely (white).

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TEXT PREPARED BY
MIT SPECIALIST

Dr. Claude Shannon, a research mathematician for Bell Telephone Laboratories, a research associate at MIT. His books include Communication theory and the recent volume "Automation Studies" on the theory of robot construction. He has prepared a paper entitled "A Symbolic Analysis of Relay and Switching Circuits" available in the GENIAC. Covers basic theory necessary for advanced circuit design, it vastly extends the range of our kit.

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Eugene Darling, Malden

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1961: Harry Olson and Herbert Belar build a sophisticated electronic machine that produced and synthesized melodices based on Markovian pitch and rhythm analysis of eleven Stephen Collins Foster songs (1961)

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### Table III. Three-note sequences of eleven Stephen Foster songs.

Probability of note following a dinote expressed in sixteenths.

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Fig. 11. Selected phrases from the output of the music composing machine. Set-up as of June 30, 1951, 4/4 time. Trinote probability derived from 11 Stephen Foster songs. Note, out of a total of 44 measures from the machine the following were selected, namely, 1 to 9, 13 to 25, 29 to 33, and 40 to 44 inclusive, and the following were ruled out, namely; 10 to 12, 26 to 28 and 34 to 39 inclusive.


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- David Zicarelli’s Jam Factory and Joel Chadabe and Zicarelli’s M (1987)
10.8. Markov Chains: Example: Shakespear

- Hamlet Act 3, Scene 1, Soliloquy
  
  YouTube (http://www.youtube.com/watch?v=-JD6gOrARk4)

- Shakespear: Hamlet: “To be or not to be”

  To be, or not to be— that is the question:  
  Whether ’tis nobler in the mind to suffer  
  The slings and arrows of outrageous fortune  
  Or to take arms against a sea of troubles,  
  And by opposing end them. To die— to sleep—  
  No more; and by a sleep to say we end  
  The heartache, and the thousand natural shocks  
  That flesh is heir to. ’Tis a consummation  
  Devoutly to be wish’d. To die— to sleep.  
  To sleep— perchance to dream: ay, there’s the rub!  
  For in that sleep of death what dreams may come  
  When we have shuffled off this mortal coil,  
  Must give us pause. There’s the respect  
  That makes calamity of so long life.  
  For who would bear the whips and scorns of time,  
  Th’ oppressor’s wrong, the proud man’s contumely,  
  The pangs of despis’d love, the law’s delay,  
  The insolence of office, and the spurns  
  That patient merit of th’ unworthy takes,  
  When he himself might his quietus make  
  With a bare bodkin? Who would these fardels bear,  
  To grunt and sweat under a weary life,  
  But that the dread of something after death—  
  The undiscover’d country, from whose bourn  
  No traveller returns— puzzles the will,  
  And makes us rather bear those ills we have  
  Than fly to others that we know not of?  
  Thus conscience does make cowards of us all,  
  And thus the native hue of resolution  
  Is sicklied o’er with the pale cast of thought,  
  And enterprises of great pith and moment  
  With this regard their currents turn awry  
  And lose the name of action.— Soft you now!  
  The fair Ophelia!— Nymph, in thy orisons  
  Be all my sins rememb’red.

- 0-order Markov re-generation

  wish’d. contumely, Be contumely, the Devoutly thus pangs by thy of fardels makes name  
  consummation pale Who we to respect coil, the to be and To Nymph, Th’ That No *Tis There’s  
  And the cowards of that When the weary or To a against wrong, And name With th’ we thought,  
  the sins That To my wrong, off perchance those Be scorns To his a that With others The quietus  
  currents fly wrong, weary that To traveller time, When have scorns wrong, pale traveller against of  
  make scorns quietus of delay, sleep. awry With to currents in and With cast coil, But have may  
  arms Th’ take arrows and The those their to regard the end we coil, fortune take

- 1-order Markov re-generation
die to others that is the name of so long life. For in the will, And enterprises of great pith and
scorns of thought, And enterprises of thought, And lose the proud man’s contumely, The
undiscover’d country, from whose bourn No more; and the whips and moment With this regard
their currents turn awry And lose the proud man’s contumely, The slings and sweat under a sleep
perchance to dream: ay, there’s the whips and scorns of office, and arrows of great pith and
scorns of something after death what dreams may come When he himself might his quietus make
With this regard their currents turn awry And makes us pause. There’s the law’s delay, The
heartache, and arrows of

• 2-order Markov re-generation

To be, or not to be wish’d. To die to sleep No more; and by a sleep to say we end The heartache,
and the thousand natural shocks That flesh is heir to. ’Tis a consummation Devoutly to be wish’d.
To die to sleep No more; and by a sleep to say we end The heartache, and the thousand natural
shocks That flesh is heir to. ’Tis a consummation Devoutly to be wish’d. To die to sleep No
more; and by a sleep to say we end The heartache, and the thousand natural shocks That flesh is
heir to. ’Tis a consummation Devoutly to be that is the question: Whether ’tis nobler in the mind
to suffer The slings and

• 3-order Markov re-generation

the name of action. Soft you now! The fair Ophelia! Nymph, in thy orisons Be all my sins
rememb’red. To be, or not to be wish’d. To die to sleep No more; and by a sleep to say we end
The heartache, and the thousand natural shocks That flesh is heir to. ’Tis a consummation
Devoutly to be that is the question: Whether ’tis nobler in the mind to suffer The slings and
arrows of outrageous fortune Or to take arms against a sea of troubles, And by opposing end
them. To die to sleep No more; and by a sleep to say we end The heartache, and the spurns That
patient merit of th’ unworthy takes, When he himself

• 4-order Markov re-generation

those ills we have Than fly to others that we know not of? Thus conscience does make cowards
of us all, And thus the native hue of resolution Is sicklied o’er with the pale cast of thought, And
enterprises of great pith and moment With this regard their currents turn awry And lose the name
of action. Soft you now! The fair Ophelia! Nymph, in thy orisons Be all my sins rememb’red. To
be, or not to be wish’d. To die to sleep No more; and by a sleep to say we end The heartache, and
the thousand natural shocks That flesh is heir to. ’Tis a consummation Devoutly to be that is the
question: Whether ’tis nobler in the

• 5-order Markov re-generation

we have shuffled off this mortal coil, Must give us pause. There’s the respect That makes calamity
of so long life. For who would bear the whips and scorns of time, Th’ oppressor’s wrong, the
proud man’s contumely, The pangs of despis’d love, the law’s delay, The insolence of office, and
the spurns That patient merit of th’ unworthy takes, When he himself might his quietus make
With a bare bodkin? Who would these fardels bear, To grunt and sweat under a weary life, But
that the dread of something after death The undiscover’d country, from whose bourn No
traveller returns puzzles the will, And makes us rather bear those ills we have Than fly to others that we know

10.9. Markov Chains: Example: Mozart Symphony 40

- Audio: Mozart: Symphony 40
- Pitch and rhythm based Markov regeneration at various orders
- Markov-generated examples [markovMozart.py]

10.10. Markov Analysis and Generation with athenaCL Python
Libraries: Text

- Use the athenaCL Markov module
- Create a markov.Transition instances to do analysis
- Example: string data [markovShakespear.py]

```python
import random
from athenaCL.libATH import markov

src = """"To be, or not to be- that is the question: Whether 'tis nobler in the mind to suffer The slings and arrows of outrageous fortune Or to take arms against a sea of troubles, And by opposing end them. To die- to sleep- No more; and by a sleep to say we end The heartache, and the thousand natural shocks That flesh is heir to. 'Tis a consummation Devoutly to be wish'd. To die- to sleep. To sleep- perchance to dream: ay, there's the rub! For in that sleep of death what dreams may come When we have shuffled off this mortal coil, Must give us pause. There's the respect That makes calamity of so long life. For who would bear the whips and scorns of time, Th' oppressor's wrong, the proud man's contumely, The pangs of despis'd love, the law's delay, The insolence of office, and the spurns That patient merit of th' unworthy takes, When he himself might his quietus make To grunt and sweat under a weary life, But that the dread of something after death- The undiscover'd country, from whose bourn No traveller returns- puzzles the will, And makes us rather bear those ills we have Than fly to others that we know not of?"
```

110
Thus conscience does make cowards of us all,
And thus the native hue of resolution
Is sicklied o'er with the pale cast of thought,
And enterprises of great pith and moment
With this regard their currents turn awry
And lose the name of action.—Soft you now!
The fair Ophelia!—Nymph, in thy orisons
Be all my sins rememb'red.""

orderMax = 2  # large numbers here will take time!
mkObj = markov.Transition()
mkObj.loadString(src, orderMax)  # source and max order

for order in range(0, orderMax+1):
    print('requested order: ' + order)
    msg = []
    for x in range(120):
        val = random.random()
        msg.append(mkObj.next(val, msg, order))
    print(' '.join(msg) + '

10.11. Markov Analysis and Generation with athenaCL Python Libraries: MIDI

• Example: pitch and rhythm data [markovMozart.py]

    import os, random, sys
    from athenaCL.libATH import midiTools
    from athenaCL.libATH import osTools
    from athenaCL.libATH import pitchTools
    from athenaCL.libATH import rhythm
    from athenaCL.libATH import markov
    from athenaCL.libATH.libOrc import generalMidi
    from athenaCL.libATH.libPmtr import parameter
    from athenaCL.libATH.libPmtr import basePmtr

    OUTDIR = '/Volumes/xdisc/_scratch'
    BEATDUR = rhythm.bpmToBeatTime(128) # provide bpm value

    def getInstName(nameMatch):
        for name, pgm in generalMidi.gmProgramNames.items():
            if name.lower().startswith(nameMatch.lower()):
                return pgm # an integer
        return None

    def convertPitch(src, octShift):
        post = []
        for pitch in src:
            midiPs = pitchTools.psToMidi(pitchTools.psNameToPs(pitch))
            midiPs = midiPs + (12*octShift)
            post.append(midiPs)
        return post # a list of integers

    def convertRhythm(src, scale):
        post = []
        for rhythm in src:
            post.append(rhythm*scale)
        return post # a list of integers

    def mozartMarkov(events, order, octaveShift, rhythmScale):
        pitchSequence = [
rhythmSequence = [.5, .5, 1, .5, .5, 1, 1,
.5, .5, 1, .5, 1, .5, .5, 1, 1,
.5, .5, 1, .5, 1, .5, 1, 2,
.5, .5, 1, .5, 1, .5, .5, 1, 2,
.5, .5, 1, 1, 1, 1, 1, 2,
.5, .5, 1, 1, 1, 1, .5, .5, .5, 1, 2,
4, 4, 3,
.5, .5, 3, .5, .5, 3,
.5, .5, 1, .5, .5, 1, .5, .5, 1]

mkPitch = markov.Transition()
mkRhythm = markov.Transition()
mkPitch.loadList(convertPitch(pitchSequence, octaveShift), order)
mkRhythm.loadList(convertRhythm(rhythmSequence, rhythmScale), order)

pitchHistory = []
rhythmHistory = []

ampGen = parameter.factory(['ws', 'e', 4, 0, 100, 120])  # sine osc b/n 90 and 120
f = random.choice(range(50, 70))
phase = random.random()
panGen = parameter.factory(['ws', 'e', f, phase, 20, 107])
score = []
tStart = 0.0

for i in range(events):
    pitch = mkPitch.next(random.random(), pitchHistory, order)
pitchHistory.append(pitch)
    rhythm = mkRhythm.next(random.random(), rhythmHistory, order)
rhythmHistory.append(rhythm)
    dur = BEATDUR * rhythm
    amp = int(round(ampGen(0)))
    pan = int(round(panGen(0)))
    event = [tStart, dur, amp, pitch, pan]
score.append(event)
tStart += dur
return score

def main(order):
    trackList = []
score = mozartMarkov(100, order, -1, 1)
    trackList.append(['part-a', getInstName('piano'), None, score])
    path = os.path.join(OUTDIR, 'test.midi')
    mObj = midiTools.MidiScore(trackList)
    mObj.write(path)  # writes in cwd
    osTools.openMedia(path)

if __name__ == '__main__':
    if len(sys.argv) != 2:
        print('args: order')
    else:
        main(int(sys.argv[1]))
10.12. Reading: Ariza: Beyond the Transition Matrix: A Language-Independent, String-Based Input Notation for Incomplete, Multiple-Order, Static Markov Transition Values


- What are some potential advantages of the transition string over the transition matrix?

- Why might modulating Markov order be desirable?

10.13. Utility Markov Analysis and Generation within athenaCL

- AUma command can be used to get an analysis string for an space-separated sequence

```
:: auma
maximum analysis order: 1
enter space-separated string: 0 1 1 1 0 1 2 3 4 0 0 2 1 3 2 4 0 0
AthenaUtility Markov Analysis
a{0}b{1}c{2}d{3}e{4}:{a=6|b=6|c=3|d=2|e=2}a:{a=3|b=2|c=1}b:{a=1|b=3|c=1|d=1}c:{b=1|d=1|e=1}d:{c=1|e=1}e:{a=2}
```

- AUmg command can be used to use a transition string to generate values

```
:: aumg
number of generations: 20
desired order: 1
enter Markov transition string:
a{0}b{1}c{2}d{3}e{4}:{a=6|b=6|c=3|d=2|e=2}a:{a=3|b=2|c=1}b:{a=1|b=3|c=1|d=1}c:{b=1|d=1|e=1}d:{c=1|e=1}e:{a=2}
AthenaUtility Markov Generator
4,0,1,1,1,1,1,3,2,1,1,1,1,2,4,0,0,1,0
```

10.14. Markov-Based Proportional Rhythm Generation

- The MarkovPulse Generator permits specifying proportional rhythms (pulse truples) as Markov states

```
:: tpv markovpulse
Rhythm Generator ParameterObject
{name,documentation}
MarkovPulse
markovPulse, transitionString, parameterObject
Description: Produces Pulse sequences by means of a Markov transition string specification and a dynamic transition order generator. The Markov transition string must define symbols that specify valid Pulses. Markov transition order is specified by a ParameterObject that produces values between 0 and the maximum order available in the Markov transition string. If generated-orders are greater than those available, the largest available transition order will be used. Floating-point order values are treated as probabilistic weightings: for example, a transition of 1.5
```
offers equal probability of first or second order selection. Arguments: (1) name, (2) transitionString, (3) parameterObject {order value}

- Command sequence:
  - emo mp
  - tin a 64
  - *simple zero-order selection*
    
    tic r mp,a{4,1}b{4,3}c{4,5}d{4,7}:\{a=4\,b=3\,c=2\,d=1\}

- *first order generation that encourages movement toward the shortest duration*
  
  tic r mp,a{8,1}b{4,3}c{4,7}d{4,13}a:{a=9\,d=1}\,b:{a=5\,c=1}\,c:{b=1}\,d:{c=1},(c,1)

- cln; clh

### 10.15. Markov-Based Value Generation

- The MarkovValue Generator permits specifying any value as Markov states, and dynamically moving between different Markov orders

:: tpv mv
Generator ParameterObject
{name,documentation}
MarkovValue markovValue, transitionString, parameterObject
Description: Produces values by means of a Markov transition string specification and a dynamic transition order generator. Markov transition order is specified by a ParameterObject that produces values between 0 and the maximum order available in the Markov transition string. If generated-orders are greater than those available, the largest available transition order will be used. Floating-point order values are treated as probabilistic weightings: for example, a transition of 1.5 offers equal probability of first or second order selection. Arguments: (1) name, (2) transitionString, (3) parameterObject {order value}

:: tpmap 100
mv,a\{2\}b\{4\}c\{7\}d\{9\}e\{11\}:{a=1|b=3|c=1|d=3|e=1}a:{a=9|e=1}b:{a=3|c=1}c:{b=3|d=1}d:{c=3|e=1}e:{d=1}, (constant, 1)
markovValue, a\{2\}b\{4\}c\{7\}d\{9\}e\{11\}:{a=1|b=3|c=1|d=3|e=1}a:{a=9|e=1}b:{a=3|c=1}c:{b=3|d=1}d:{c=3|e=1}e:{d=1}, (constant, 1)
TPmap display complete.
The modulating the order of the Markov chain can create dynamic long-range behavior

:: tpmap 100
mv,a{2}b{4}c{7}d{9}e{11}:{a=1|b=3|c=1|d=3|e=1}a:{a=9|e=1}b:{a=3|c=1}c:{b=3|d=1}d:{c=3|e=1}e:{d=1}, (wp,e,50,0,1,0)
markovValue, a{2}b{4}c{7}d{9}e{11}:{a=1|b=3|c=1|d=3|e=1}a:{a=9|e=1}b:{a=3|c=1}c:{b=3|d=1}d:{c=3|e=1}e:{d=1},

TPmap display complete.

Command sequence:

- emo m
- tin a 26
- rhythm generated with absolute values via ConvertSecond and a dynamic WaveHalfPeriodSine generator
tie r cs,(whps,e,(bg,rp,(5,10,15,20)),0,.200,.050)

- first-order selection
tie f
mv,a{2}b{4}c{7}d{9}e{11}:{a=1|b=3|c=1|d=3|e=1}a:{a=9|e=1}b:{a=3|c=1}c:{b=3|d=1}d:{c=3|e=1}e:{d=1},(c,1)

- dynamic first and zero order selection
tie f
mv,a{2}b{4}c{7}d{9}e{11}:{a=1|b=3|c=1|d=3|e=1}a:{a=9|e=1}b:{a=3|c=1}c:{b=3|d=1}d:{c=3|e=1}e:{d=1},(wp,e,100,0,1,0)

- zero-order Markov amplitude values
tie a \( mva \{a=6|b=4|c=3|d=1\} \)

- amplitude values scaled by a dynamic WaveHalfPeriodPulse

\[
tie a om,(mva\{a=6|b=4|c=3|d=1\},(whpp,\epsilon,(bg,rp,(5,15,10))))
\]

- octave values are provided by a first-order Markov chain

\[
tie o mva\{a=9|d=1\}b\{a=3|b=1\}c\{b=3|c=1\}d\{c=1\},(c,1)
\]

- tie t 0,60

- cln; clh

10.16. Markov-Based Combined Analysis and Generation

- The MarkovGeneratorAnalysis Generator permits using the output of a ParameterObject as the source for Markov analysis

:: tpv mga
Generator ParameterObject
{name,documentation}
MarkovGeneratorAnalysis markovGeneratorAnalysis, parameterObject, valueCount, maxAnalysisOrder, parameterObject
Description: Produces values by means of a Markov analysis of values provided by a source Generator ParameterObject; the analysis of these values is used with a dynamic transition order Generator to produce new values. The number of values drawn from the source Generator is specified with the valueCount argument. The maximum order of analysis is specified with the maxAnalysisOrder argument. Markov transition order is specified by a ParameterObject that produces values between 0 and the maximum order available in the Markov transition string. If generated-orders are greater than those available, the largest available transition order will be used. Floating-point order values are treated as probabilistic weightings: for example, a transition of 1.5 offers equal probability of first or second order selection. Arguments: (1) name, (2) parameterObject {source Generator}, (3) valueCount, (4) maxAnalysisOrder, (5) parameterObject {output order value}

- First order analysis and regeneration of a sine oscillation

:: tpmap 100 mga,(ws,e,30),30,2,(c,1)
markovGeneratorAnalysis, (waveSine, event, (constant, 30), 0, (constant, 0), (constant, 1)), 30, 2, (constant, 1)
TPmap display complete.
• Analysis and regeneration of a sine oscillation with dynamic orders from 0.5 to 1.5

Floating-point orders are treated as probabilistic weightings toward nearest integers

:: tmap 100 mga,(ws,e,30),30,2,(ws,e,50,0,0.5,1.5)
markovGeneratorAnalysis, (waveSine, event, (constant, 30), 0, (constant, 0),
(constant, 1)), 30, 2, (waveSine, event, (constant, 50), 0, (constant, 0.5),
(constant, 1.5))
TPmap display complete.

10.17. Resuming PD Tutorial

• PD Tutorial