Quantum Physics I (8.04) Spring 2016
Assignment 7

MIT Physics Department
April 1, 2016

Due Friday April 8, 2016
12:00 noon

Reading: Griffiths sections 2.5 and 2.3.

Problem Set 7

1. Two delta functions [15 points]
   Consider a particle of mass \(m\) moving in a one-dimensional double well potential
   \[ V(x) = -g\delta(x-a) - g\delta(x+a), \quad g > 0. \]
   (a) Find transcendental equations for the bound state energy eigenvalues of the system. Plot the energy levels in units of \(\hbar^2/(ma^2)\) as a function of the dimensionless parameter \(\lambda \equiv mag/\hbar^2\). Explain the features of the plot.
   (b) In the limit of large separation \(2a\) between the wells find a simple formula for the splitting between the ground state and the first excited state.

2. Sketching wavefunctions. Griffiths 2.47, p. 87. [10 points]
   In this problem you should try to figure out intuitively how the solutions look. It is a good idea then to check your intuition with the shooting method and the setup of the \(H^+_2\) ion.

3. Harmonic oscillators beyond the turning points [10 points]
   For the simple harmonic oscillator energy eigenstates with \(n = 0, 1,\) and \(2\), calculate the probability that the coordinate \(x\) takes a value greater than the amplitude of a classical oscillator of the same energy.

4. Harmonic oscillator computations [15 points]
   (a) Calculate the expectation value of \(x^4\) on the energy eigenstate with number \(n\).
   (b) Calculate \(\Delta x\) and \(\Delta p\) on the energy eigenstate with number \(n\). What is the value of the product \(\Delta x\Delta p\)?
(c) Consider the polynomials $H_n(\xi)$ defined by the generating function

$$e^{-s^2+2s\xi} = \sum_{n=0}^{\infty} H_n(\xi) \frac{s^n}{n!}.$$ 

Verify that $H_n(\xi) = (2\xi)^n + \ldots$ where the dots represent terms with lower powers of $\xi$. Show that the polynomials $H_n(\xi)$ so defined satisfy the Hermite differential equation:

$$H_n'' - 2\xi H_n' + 2nH_n = 0.$$

5. **Harmonic oscillator and a wall.** Griffiths Problem 2.42. p. 86. [5 points]

6. **Harmonic oscillator oscillating!** [10 points]

A particle of mass $m$ in a harmonic oscillator with frequency $\omega$ has an initial, time zero wavefunction

$$\Psi(x, 0) = \frac{1}{\sqrt{2}} (\varphi_0(x) + \varphi_1(x)),$$

where $\varphi_0$ and $\varphi_1$ are the normalized eigenstates of the Hamiltonian with number eigenvalue zero and one, respectively.

(a) Write down $\Psi(x, t)$ and $|\Psi(x, t)|^2$. You may leave your expressions in terms of $\varphi_0$ and $\varphi_1$.

(b) Find $\langle x \rangle$ as a function of time. What is the amplitude of this oscillation and what is its frequency?

(c) Find $\langle p \rangle$ as a function of time.

(d) Show that for any harmonic oscillator state, the probability distribution $|\Psi(x, t)|^2$ is equal to $|\Psi(x, t + T)|^2$ for $T = \frac{2\pi}{\omega}$. 
