1. Radiation pressure and the Eddington limit.

(a) Show that the condition that an optically thin cloud of material can be ejected by radiation pressure from a nearby luminous object is that the mass to luminosity ratio \( M/L \) for the object be less than \( \kappa/(4\pi G c) \), where \( \kappa \) is the mass absorption coefficient (assumed to be independent of frequency). (Hint: The force per unit mass due to radiation pressure on absorbing material is \( \int (\kappa \nu F_\nu/c) d\nu \), where \( F_\nu \) is the radiative flux per unit frequency.)

(b) Calculate the terminal velocity \( v \) attained by such a cloud under radiation and gravitational forces alone, if it starts from rest at a distance \( R \) from the object. Show that

\[
\frac{v^2}{2} = \frac{2GM}{R} \left( \frac{\kappa L}{4\pi G M c} - 1 \right).
\]

(c) A minimum value for \( \kappa \) may be estimated for pure hydrogen as that due to Thomson scattering off free electrons, when the hydrogen is completely ionized. The Thomson cross-section is \( \sigma_T = 6.65 \times 10^{-25} \) cm\(^2\). The mass scattering coefficient is thus \( \sigma_T/m_H \), where \( m_H \) is the mass of a hydrogen atom. Show that the maximum luminosity that a central mass \( M \) can have and still not spontaneously eject hydrogen by radiation pressure is

\[
L_{\text{Edd}} = \frac{4\pi G M m_H}{\sigma_T} = 1.3 \times 10^{38} (M/M_\odot) \text{ erg s}^{-1}.
\]

This is called the Eddington limit.

2. Stability against convection.

(a) In lecture, we derived the following condition for stability against convection in a star,

\[
\frac{\rho}{\gamma P} \frac{dP}{dr} - \frac{d\rho}{dr} > 0,
\]

where \( P \) is the pressure, \( \rho \) is the density, \( \gamma \) is the exponent in the adiabatic equation of state \( P = K \rho^\gamma \), and \( r \) is distance from the stellar center. Using the ideal gas law \( P = \rho kT/\mu m_p \), show that this condition can also be written as

\[
\frac{dT}{dr} > \left( 1 - \frac{1}{\gamma} \right) \frac{T}{P} \frac{dP}{dr}.
\]

Furthermore, use the relevant stellar structure equations (for a radiative star) to show that this reduces to

\[
L(r) < \left( 1 - \frac{1}{\gamma} \right) \frac{16\pi a c T^4 G M(r)}{3\kappa P},
\]

where \( L(r) \) and \( M(r) \) are the luminosity and enclosed mass at radius \( r \), \( \kappa \) is the opacity, and \( a \) is the radiation constant.
Show that to avoid convection in a stellar region where the equation of state is that of an ideal monatomic gas, the luminosity at a given radius must be limited by (in cgs units)

\[ L(r) < 1.22 \times 10^{-18} \mu T^3 \left( \frac{\kappa \rho}{\kappa \rho + \mu} \right) M(r) \]

where \( T(r) \) is the temperature, \( \mu \) is the mean molecular weight, \( \kappa \) is the Rosseland mean opacity, and \( M(r) \) is the mass enclosed at a radius \( r \).

3. **Radiative transfer.** HK&T, Problem 4.1.

4. **Helium ionization.** HK&T, Problem 4.6. (Note that this problem refers back to HK&T Problem 3.1, which is essentially Problem 3 from our Problem Set 2.)

5. **Stimulated emission.** HK&T, Problem 4.9.