**PROCESS CAPABILITY**

GIVEN: A VARIABLE TO BE TRACKED, For example, the diameter of a machined part:

Let $T=$ Target Diameter

- $USL=$ Upper Specification Limit
- $LSL=$ Lower Specification Limit

Collect some data for the diameters, $X_1, \ldots, X_n$

Suppose $\mu=\frac{\sum X_i}{n}$, $\sigma=\frac{\sum (X_i-\mu)^2}{n-1}$.

How well is the process performing?
One Measure: $C_p=(USL-LSL)/6\sigma$

If $\mu=T$, $\mu+3\sigma=USL$, and $\mu-3\sigma=LSL$, then, $C_p=1$. 

$\mu+3\sigma$ | . . . . . .
---|---
$\mu$ | . . . . . . . . . .
---|---
$\mu-3\sigma$ | . . . . . . . . . .
If \( \mu = T \), \( \mu + 6\sigma = \text{USL} \), and \( \mu - 6\sigma = \text{LSL} \), then, \( C_p = 2 \).

Which is better, \( C_p = 1 \) or \( C_p = 2 \)?
Define \( Cpk = \min\{(USL-\mu)/3\sigma, (\mu-LSL)/3\sigma\} \)

Suppose \( T+3\sigma = USL \) and \( T-3\sigma = LSL \).

If \( \mu = T \), then \( Cp = Cpk \)

If \( \mu > T \), e.g., if \( \mu = T+\sigma \), then

\[
Cp = (T+3\sigma - T-3\sigma)/6\sigma = 1
\]

\[
Cpk = \min\{(T+3\sigma - T-\sigma)/3\sigma, (T+\sigma - T+3\sigma)/3\sigma\} = \min\{2/3, 4/3\} = 2/3.
\]

Which measure do you prefer?

At Motorola, 6\( \sigma \) program aims for \( Cpk = 2 \) for all processes. What defect frequency does that imply? (Hint: \( Z \sim N(0,1) = 6 \Rightarrow \text{area under curve} = .4999983. \))