Load Flow I used the decoupled Newton-Raphson method for this, but anything that you could get running should do. The purpose of this problem was simply to make use of the routine you spent so much effort getting up and running. You might recall the voltages and line currents resulting from the load flow, with everything operating normally, were:

<table>
<thead>
<tr>
<th>Bus</th>
<th>Per-Unit</th>
<th>Angle (Radians)</th>
<th>(Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>1.0020</td>
<td>0.0072</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>0.9896</td>
<td>-0.0160</td>
<td>-0.91</td>
</tr>
<tr>
<td>4</td>
<td>0.9545</td>
<td>-0.0918</td>
<td>-5.26</td>
</tr>
<tr>
<td>5</td>
<td>0.9548</td>
<td>-0.0909</td>
<td>-5.21</td>
</tr>
<tr>
<td>6</td>
<td>0.9574</td>
<td>-0.0876</td>
<td>-5.02</td>
</tr>
<tr>
<td>7</td>
<td>0.9516</td>
<td>-0.0972</td>
<td>-5.57</td>
</tr>
<tr>
<td>8</td>
<td>0.9640</td>
<td>-0.0732</td>
<td>-4.19</td>
</tr>
<tr>
<td>9</td>
<td>0.9657</td>
<td>-0.0603</td>
<td>-3.46</td>
</tr>
<tr>
<td>10</td>
<td>0.9073</td>
<td>-0.1325</td>
<td>-7.59</td>
</tr>
<tr>
<td>11</td>
<td>0.9745</td>
<td>-0.0505</td>
<td>-2.90</td>
</tr>
<tr>
<td>12</td>
<td>0.9749</td>
<td>-0.0463</td>
<td>-2.65</td>
</tr>
<tr>
<td>13</td>
<td>0.8981</td>
<td>-0.1556</td>
<td>-8.91</td>
</tr>
<tr>
<td>14</td>
<td>0.9961</td>
<td>-0.0083</td>
<td>-0.47</td>
</tr>
<tr>
<td>15</td>
<td>0.9609</td>
<td>-0.0734</td>
<td>-4.21</td>
</tr>
<tr>
<td>16</td>
<td>0.9547</td>
<td>-0.0859</td>
<td>-4.92</td>
</tr>
<tr>
<td>17</td>
<td>0.9582</td>
<td>-0.0746</td>
<td>-4.28</td>
</tr>
</tbody>
</table>

Line Current Flows

<table>
<thead>
<tr>
<th>Line</th>
<th>Amperes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42.8</td>
</tr>
<tr>
<td>2</td>
<td>192.1</td>
</tr>
<tr>
<td>3</td>
<td>88.6</td>
</tr>
<tr>
<td>4</td>
<td>370.1</td>
</tr>
<tr>
<td>5</td>
<td>358.8</td>
</tr>
<tr>
<td>6</td>
<td>215.6</td>
</tr>
<tr>
<td>7</td>
<td>281.7</td>
</tr>
<tr>
<td>8</td>
<td>367.8</td>
</tr>
<tr>
<td>9</td>
<td>213.7</td>
</tr>
<tr>
<td>10</td>
<td>12.9</td>
</tr>
<tr>
<td>11</td>
<td>136.1</td>
</tr>
<tr>
<td>12</td>
<td>46.4</td>
</tr>
</tbody>
</table>
Note that the voltage on Bus 10 is a bit low (if you want voltage to be within 5% of nominal). But the other voltages are all good. Now we try some bad things:

1. Loss of the transformer between buses 9 and 17. To do this we simply set the elements of the node incidence matrix to zero, taking that branch out of the network. the resulting voltages and currents are:

<table>
<thead>
<tr>
<th>Bus</th>
<th>Per-Unit</th>
<th>Angle (Radians)</th>
<th>Angle (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.9908</td>
<td>-0.0260</td>
<td>1.49</td>
</tr>
<tr>
<td>3</td>
<td>0.9682</td>
<td>-0.0969</td>
<td>5.55</td>
</tr>
<tr>
<td>4</td>
<td>0.9383</td>
<td>-0.1001</td>
<td>5.73</td>
</tr>
<tr>
<td>5</td>
<td>0.9394</td>
<td>-0.0977</td>
<td>5.60</td>
</tr>
<tr>
<td>6</td>
<td>0.9309</td>
<td>-0.1095</td>
<td>6.27</td>
</tr>
<tr>
<td>7</td>
<td>0.9477</td>
<td>-0.0819</td>
<td>4.69</td>
</tr>
<tr>
<td>8</td>
<td>0.9648</td>
<td>-0.0558</td>
<td>3.20</td>
</tr>
<tr>
<td>9</td>
<td>0.6952</td>
<td>-0.3647</td>
<td>20.90</td>
</tr>
<tr>
<td>10</td>
<td>0.9633</td>
<td>-0.0560</td>
<td>3.21</td>
</tr>
<tr>
<td>11</td>
<td>0.9589</td>
<td>-0.0542</td>
<td>3.11</td>
</tr>
<tr>
<td>12</td>
<td>0.7405</td>
<td>-0.3150</td>
<td>18.05</td>
</tr>
<tr>
<td>13</td>
<td>0.9890</td>
<td>-0.0108</td>
<td>0.62</td>
</tr>
<tr>
<td>14</td>
<td>0.9309</td>
<td>-0.0920</td>
<td>5.27</td>
</tr>
<tr>
<td>15</td>
<td>0.9090</td>
<td>-0.1234</td>
<td>7.07</td>
</tr>
<tr>
<td>16</td>
<td>0.6952</td>
<td>-0.3647</td>
<td>20.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Line</th>
<th>Amperes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.4</td>
</tr>
<tr>
<td>2</td>
<td>226.4</td>
</tr>
<tr>
<td>3</td>
<td>71.2</td>
</tr>
<tr>
<td>4</td>
<td>372.2</td>
</tr>
<tr>
<td>5</td>
<td>341.4</td>
</tr>
<tr>
<td>6</td>
<td>300.0</td>
</tr>
<tr>
<td>7</td>
<td>315.1</td>
</tr>
</tbody>
</table>
As expected, the current in lines 21 and 17 both go to zero. Line 19 is quite a bit more heavily loaded, and the voltages on buses 10 and 13 are substantially below normal. It appears this transformer is necessary for normal system operation.

2. An increase in the load on bus 10 from 15 to 35 MW,

<table>
<thead>
<tr>
<th>Bus</th>
<th>Voltages</th>
<th>Angle (Radians)</th>
<th>(Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.9992</td>
<td>0.0025</td>
<td>0.14</td>
</tr>
<tr>
<td>3</td>
<td>0.9851</td>
<td>-0.0242</td>
<td>-1.39</td>
</tr>
<tr>
<td>4</td>
<td>0.9501</td>
<td>-0.1003</td>
<td>-5.75</td>
</tr>
<tr>
<td>5</td>
<td>0.9506</td>
<td>-0.0991</td>
<td>-5.68</td>
</tr>
<tr>
<td>6</td>
<td>0.9531</td>
<td>-0.0959</td>
<td>-5.50</td>
</tr>
<tr>
<td>7</td>
<td>0.9470</td>
<td>-0.1061</td>
<td>-6.08</td>
</tr>
<tr>
<td>8</td>
<td>0.9600</td>
<td>-0.0807</td>
<td>-4.63</td>
</tr>
<tr>
<td>9</td>
<td>0.9613</td>
<td>-0.0694</td>
<td>-3.97</td>
</tr>
<tr>
<td>10</td>
<td>0.8867</td>
<td>-0.1907</td>
<td>-10.92</td>
</tr>
<tr>
<td>11</td>
<td>0.9716</td>
<td>-0.0556</td>
<td>-3.19</td>
</tr>
<tr>
<td>12</td>
<td>0.9711</td>
<td>-0.0532</td>
<td>-3.05</td>
</tr>
<tr>
<td>13</td>
<td>0.8821</td>
<td>-0.1972</td>
<td>-11.30</td>
</tr>
<tr>
<td>14</td>
<td>0.9936</td>
<td>-0.0109</td>
<td>-0.62</td>
</tr>
<tr>
<td>15</td>
<td>0.9556</td>
<td>-0.0840</td>
<td>-4.81</td>
</tr>
<tr>
<td>16</td>
<td>0.9485</td>
<td>-0.1006</td>
<td>-5.76</td>
</tr>
<tr>
<td>17</td>
<td>0.9522</td>
<td>-0.0922</td>
<td>-5.28</td>
</tr>
</tbody>
</table>

Line Current Flows

<table>
<thead>
<tr>
<th>Line</th>
<th>Amperes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>56.1</td>
</tr>
<tr>
<td>2</td>
<td>211.7</td>
</tr>
<tr>
<td>3</td>
<td>75.5</td>
</tr>
<tr>
<td>4</td>
<td>371.4</td>
</tr>
</tbody>
</table>
Nothing really bad here, except for a somewhat lower voltage on bus 10.

3. A combination of the previous two conditions. As it turns out, the system won’t handle this combination. Below is what happens with the largest loading on Bus 10 I could find: 23.5 MW. For larger attempted loading the voltage on Bus 10 collapses.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Per-Unit</th>
<th>Angle (Radians)</th>
<th>(Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>0.0000</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.9781</td>
<td>-0.0004</td>
<td>-0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.9450</td>
<td>-0.0327</td>
<td>-1.87</td>
</tr>
<tr>
<td>4</td>
<td>0.9267</td>
<td>-0.1044</td>
<td>-5.98</td>
</tr>
<tr>
<td>5</td>
<td>0.9189</td>
<td>-0.1087</td>
<td>-6.23</td>
</tr>
<tr>
<td>6</td>
<td>0.9187</td>
<td>-0.1067</td>
<td>-6.11</td>
</tr>
<tr>
<td>7</td>
<td>0.9078</td>
<td>-0.1199</td>
<td>-6.87</td>
</tr>
<tr>
<td>8</td>
<td>0.9289</td>
<td>-0.0896</td>
<td>-5.13</td>
</tr>
<tr>
<td>9</td>
<td>0.9555</td>
<td>-0.0595</td>
<td>-3.41</td>
</tr>
<tr>
<td>10</td>
<td>0.5314</td>
<td>-0.5882</td>
<td>-33.70</td>
</tr>
<tr>
<td>11</td>
<td>0.9504</td>
<td>-0.0608</td>
<td>-3.48</td>
</tr>
<tr>
<td>12</td>
<td>0.9407</td>
<td>-0.0607</td>
<td>-3.48</td>
</tr>
<tr>
<td>13</td>
<td>0.6051</td>
<td>-0.4521</td>
<td>-25.90</td>
</tr>
<tr>
<td>14</td>
<td>0.9821</td>
<td>-0.0127</td>
<td>-0.73</td>
</tr>
<tr>
<td>15</td>
<td>0.8998</td>
<td>-0.1050</td>
<td>-6.02</td>
</tr>
<tr>
<td>16</td>
<td>0.8636</td>
<td>-0.1491</td>
<td>-8.54</td>
</tr>
<tr>
<td>17</td>
<td>0.5314</td>
<td>-0.5882</td>
<td>-33.70</td>
</tr>
</tbody>
</table>

Line Current Flows

<table>
<thead>
<tr>
<th>Line</th>
<th>Amperes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>97.6</td>
</tr>
</tbody>
</table>
Voltage This one turns out to be a bit more difficult than one would think. I found it expedient to use a kind of decoupled Newton-Raphson method. Noting that:

\[ p = \frac{V V_{\infty} \sin \theta}{x_\ell} \]
\[ q = \frac{V V_{\infty} \cos \theta - V_{\infty}^2}{x_\ell} \]

we can, for some set of \( V \) and \( \theta \), compute values of \( p \) and \( q \). The trick is to get values of \( V \) and \( \theta \) that result in the right values of \( p \) and \( q \). We can generate corrections to \( V \) and \( \theta \) by noting that:

\[ \frac{dp}{d\theta} = \frac{V V_{\infty} \cos \theta}{x_\ell} \]
\[ \frac{dq}{dV} = \frac{V_{\infty} \cos \theta}{x_\ell} \]
Then it is straightforward to start with some guess and do the corrections until an arbitrary error is reached. If the objective for real and reactive power are \( p_r \) and \( q_r \), at each step:

\[
V_{\text{new}} = V_{\text{old}} - \frac{q_r - q v}{\frac{dq}{dv}} \\
\theta_{\text{new}} = \theta_{\text{old}} - \frac{p_r - p v}{\frac{dp}{d\theta}}
\]

This is implemented in an appended Matlab script. Terminal voltage is shown in Figure 2.

![Figure 2: Terminal Voltage](image)

To find internal voltage, note that, if we hold terminal voltage to be ‘real’, we find current by:

\[
i = \frac{p - jq}{v}
\]

Then the (complex) value of internal voltage is:

\[
e_{af} = v + j * xd * i
\]

This is shown in Figure 3. Note that the curve corresponding to zero real power has a kink in it, corresponding to a zero crossing. This machine would not operate stably at large negative reactive power, so not all of these points are realistic.

**Equivalent Impedance** This one, too, turns out to be a bit trickier than it looks at first. Note that current into the 'swing bus' (Bus 1) is:

\[
i_1 = y_{1,1}v_1 + y_{2,2}v_2 + ...
\]
The thevenin equivalent voltage is the voltage that would exist at that bus when the current is zero, assuming all other voltages are the same as in the actual situation. Thus, if we know all of the other voltages, the thevenin equivalent voltage is:

\[ v_{\text{thev}} = \frac{1}{y_{1,1}} \sum_{k=2}^{N} y_{1,k} v_k \]

Of course the thevenin equivalent admittance is just \( y_{1,1} \). There are other ways of calculating it, including noting that the current times driving point impedance (reciprocal of admittance) is also the difference between bus voltage and thevenin equivalent voltage. The calculations are shown in an appended script and the results are:

**Swing Bus**
- Real Power = 1.54894
- Reactive Power = 0.583641
- Current at Bus 1 = 1.54894 + j -0.583641
- Driving Point Admittance at Bus 1 = 6.36863 + j -36.0464
- Thevenin Equivalent Voltage = 0.976937 + j -0.038896
- Check: Real Power = 1.54894 Reactive Power = 0.583641
% 6.691 Problem Set 3, Problem 2

V_infty = 1.0;  % basic data
xl = .1;
xd = 2;
pv = [0 .5 1.0];  % stated range
qv = -.5:.01:.5;
V = zeros(length(pv), length(qv));
D = zeros(length(pv), length(qv));
tol = 1e-8;

for i = 1:length(pv)
    pr = pv(i);
    v = 1;  % we start here
    delt = 0;
    for k = 1:length(qv)
        qr = qv(k);
        not_done = 1;
        while not_done == 1
            dpdd = v*V_infty*cos(delt)/xl;
            dqdv = V_infty*cos(delt)/xl;
            p = v*V_infty*sin(delt)/xl;
            q = (v*V_infty*cos(delt)-V_infty^2)/xl;
            E = (p-pr)^2 + (q-qr)^2;
            if E < tol
                not_done = 0;
                break
            end
            delt = delt - (p-pr)/dpdd;
            v = v - (q-qr)/dqdv;
        end  % end of while loop
        V(i, k) = v;
        D(i, k) = delt;
    end  % end of q loop
end  % end of p loop

figure(1)
clf
hold on
for i = 1:length(pv)
    plot(qv, V(i, :))
end
hold off
title('Problem 3.2')
ylabel('Bus Voltage')
xlabel('Reactive Power Q')

% now let's just check to see if this is right
Pchk = zeros(length(pv), length(qv));
Qchk = zeros(length(pv), length(qv));
for i = 1:length(pv)
    for k = 1:length(qv)
        Pchk(i, k) = V(i, k)*V_infty*sin(D(i, k))/xl;
        Qchk(i, k) = (V(i, k)*V_infty*cos(D(i, k))-V_infty^2)/xl;
    end
end

figure(2)
clf
hold on
for i = 1:length(pv)
    plot(Qchk(i, :), Pchk(i, :))
end
hold off
title('Problem 3.2')
ylabel('Real Power')
xlabel('Reactive Power')

% finally, get eaf
eaf = zeros(length(pv), length(qv));
for i = 1:length(pv)
    for k = 1:length(qv)
        I = (pv(i) -j*qv(k))/V(i, k);
        eaf(i, k) = abs(V(i, k) + j*I*xd);
    end
end

figure(3)
clf
hold on
for i = 1:length(pv)
    plot(qv, eaf(i, :))
end
hold off
title('Problem 3.2')
ylabel('Internal Voltage eaf')
xlabel('Reactive Power')

9
Solution to Problem Set 2

Newton-Raphson Solution. Decoupled.
Augmented to get bus 1 equivalents


and here are the connections for each line

\[ C_1 = \begin{bmatrix} 1 & 14 \\ 14 & 2 \\ 11 & 2 \\ 1 & 9 \\ 9 & 4 \\ 11 & 5 \\ 2 & 12 \\ 5 & 8 \\ 5 & 4 \\ 5 & 7 \\ 5 & 6 \\ 12 & 3 \\ 6 & 3 \\ 7 & 15 \\ 3 & 15 \\ 17 & 10 \\ 10 & 13 \\ 13 & 16 \\ 8 & 12 \\ 9 & 17 \\ 15 & 16 \end{bmatrix} \]

\[ Pb = 100e6; \]  
we are going to use this base power

\[ Vb1 = 161e3; \]  
and these base voltages

\[ Vb2 = 69e3; \]

\[ Zb1 = Vb1^2/Pb; \]

\[ Zb2 = Vb2^2/Pb; \]

\[ Ib1 = Pb/(sqrt(3)*Vb1); \]

\[ Ib2 = Pb/(sqrt(3)*Vb2); \]
\[ z_\perunit = \text{zeros(size}(Z_\perunit)); \]
% unfortunately we have to cobble together the impedance vector
\[ z_\perunit(1:16) = \frac{Z_\perunit(1:16)}{Zb1}; \]
\[ z_\perunit(20) = \frac{Z_\perunit(20)}{Zb1}; \]
\[ z_\perunit(17:19) = \frac{Z_\perunit(17:19)}{Zb2}; \]
% the last two lines are transformers
\[ z_\perunit = \begin{bmatrix} z_\perunit & j*0.08/1.5 & j*0.08/1.5 \end{bmatrix}; \]
\[ \text{fprintf('Line Impedances 
')} \]
\[ \text{fprintf('Buses \hspace{2cm} Ohms \hspace{2cm} Per-Unit\n')} \]
for \( i = 1:\text{length}(Z_\perunit) \)
\[ \text{fprintf('C_\perunit(i, 1), C_\perunit(i, 2), real}(Z_\perunit(i)), \text{imag}(Z_\perunit(i)),...} \]
\[ \text{real}(z_\perunit(i)), \text{imag}(z_\perunit(i)) \]
end
\[ N_\text{l} = \text{length}(z_\perunit); \]
\[ N_\text{b} = 17; \]
% Now construct the node-incidence matrix:
\[ \text{NI} = \text{zeros}(N_\text{b}, N_\text{l}); \]
for \( i = 1:N_\text{l} \)
if \( C_\perunit(i, 1) \neq 0, \)
\[ \text{NI}(C_\perunit(i, 1), i) = 1; \]
\[ \text{NI}(C_\perunit(i, 2), i) = -1; \]
end
end
\[ y_\perunit = \text{zeros}(N_\text{l}); \]
% now the line admittance matrix is:
for \( i = 1:N_\text{l} \)
\[ y_\perunit(i, i) = \frac{1}{z_\perunit(i)}; \]
end
% and the bus admittance matrix is:
\[ y_\text{bus} = \text{NI} \times y_\perunit \times \text{NI}'; \]
% Here are the bus power flows:
\[ S_\text{bus} = \begin{bmatrix} 2.2+j*0.7; \end{bmatrix}; \]
% but note we are going to ignore this one
\[ 2.2+j*0.7; \]
\[ 2.2+j*0.7; \]
\[ -0.6-j*1.1; \]
\[ -1-j*3; \]
\[ -0.8-j*15; \]

11
% now here is an initial guess about voltages:
V = ones(Nb, 1);
th = zeros(Nb, 1);

% now we go into a loop
n_iter = 0;
not_done = 1;
while (not_done == 1)
P = zeros(Nb, 1);
Q = zeros(Nb, 1);
for i = 1:Nb,
   for k = 1:Nb,
P(i) = P(i) + V(i)*V(k)*(G(i, k)*cos(th(i)-th(k))+B(i,k)*sin(th(i)-th(k)));
   Q(i) = Q(i) + V(i)*V(k)*(G(i, k)*sin(th(i)-th(k))-B(i,k)*cos(th(i)-th(k)));
   end
end

X = [th(2:Nb); V(2:Nb)]; % to be found
J11 = zeros(Nb-1); % components of the Jacobian
J12 = zeros(Nb-1);
J21 = zeros(Nb-1);
J22 = zeros(Nb-1);

for i = 2:Nb
   for k = 2:Nb
      ii= i-1;
      kk = k-1;
      if k < i
         % code here
      end
   end
end

for k = 1:Nb
   for ii = 1:Nb
      % code here
   end
end
\[
J_{11}(i, kk) = V(i) * V(k) * (G(i, k) * \sin(\theta(i) - \theta(k)) - B(i, k) * \cos(\theta(i) - \theta(k))); \\
J_{12}(i, kk) = V(i) * (G(i, k) * \cos(\theta(i) - \theta(k)) + B(i, k) * \sin(\theta(i) - \theta(k))); \\
J_{21}(i, kk) = -V(i) * V(k) * (G(i, k) * \cos(\theta(i) - \theta(k)) + B(i, k) * \sin(\theta(i) - \theta(k))); \\
J_{22}(i, kk) = V(i) * (G(i, k) * \sin(\theta(i) - \theta(k)) - B(i, k) * \cos(\theta(i) - \theta(k))); \\
\]

else
\[
J_{11}(i, ii) = -V(i)^2 * B(i, i) - Q(i); \\
J_{12}(i, ii) = P(i) / V(i) + V(i) * G(i, i); \\
J_{21}(i, ii) = P(i) - V(i)^2 * G(i, i); \\
J_{22}(i, ii) = Q(i) / V(i) - V(i) * B(i, i); \\
\]
end % end of if k = i
end % end of index k loop
end % end of index i loop: Jacobian is constructed

\[
J = [J_{11} \; \text{zeros(Nb-1)}; \; \text{zeros(Nb-1)} \; J_{22}]; \\
\]

% now find real and reactive power
\[
\text{PE} = P - P_r; \\
\text{QE} = Q - Q_r; \\
E = [\text{PE}(2:Nb); \; \text{QE}(2:Nb)]; \\
\text{SE} = \text{sum}(E .^2); \\
\]

if SE < tol
\[
\text{not_done} = 0; \\
\]
end

\[
X = X - \text{inv}(J) * E; \\
\text{th} = [0; \; X(1:Nb-1)]; \\
V = [1; \; X(Nb:2*Nb-2)]; \\
\]

n_iter = n_iter + 1;

fprintf('Error = %.g\n', SE);
%pause
end

fprintf('That took %10.0f iterations\n', n_iter)

% now generate per-unit line flows
\[
\text{v_bus} = V .* \exp(j .* \text{th}); \\
\text{v_line} = \text{NI}' * \text{v_bus}; \\
\text{i_line} = y_{\text{line}} * \text{v_line}; \\
\]

\[
\text{I_line} = \text{zeros(size(i_line))}; \\
\text{I_line}(1:16) = \text{Ib1} .* \text{i_line}(1:16); \\
\]
I_line(17:19) = Ib2 .* i_line(17:19);
I_line(20:22) = Ib1 .* i_line(20:22);

% now generate a report:

fprintf('Bus Voltages are:
')
fprintf('Bus 	 Per-Unit 	 Angle (Radians) 	 (Degrees)\n');
for i = 1:Nb
    fprintf('%3.0f 	 %3.4f 	 %8.4f 	 %6.2f\n',i, V(i), th(i), (180/pi)*th(i));
end

fprintf('Line Current Flows\n')
fprintf('Line Amperes\n')
for i = 1:22
    fprintf('%3.0f 	 %3.1f\n',i, abs(I_line(i)));
end

% now we are working on Problem Set 3: We want to get the thevenin
% equivalent impedance seen from unit 1

i_sb = i_line(1) + i_line(2) + i_line(5); % this is current into bus 1
p_sb = real(i_sb);
q_sb = - imag(i_sb);
y_thev = y_bus(1, 1)
v_thev = 1 - i_sb/y_thev;
va = conj((1-v_thev)*y_thev);

fprintf('Swing Bus
')
fprintf('Real Power = %g Reactive Power = %g
', p_sb, q_sb)
fprintf('Current at Bus 1 = %g + j %g
', real(i_sb), imag(i_sb))
fprintf('Driving Point Admittance at Bus 1 = %g + j %g
', real(y_thev), imag(y_thev))
fprintf('Thevenin Equivalent Voltage = %g + j %g
', real(v_thev), imag(v_thev));
fprintf('Check: Real Power = %g Reactive Power = %g
', real(va), imag(va));

% here is yet another way of doing the calculation
V_e = v_bus(2:length(v_bus));
Y = y_bus(1, 2:length(y_bus(1,:)));
V_thev = -sum(V_e .* Y.') / y_thev

% now check

VA = conj((1 - V_thev) * y_thev)