**Problem 3.5**

Two tanks with equal volume are connected with a Joule-Thompson valve. Tank I contains an ideal gas at 10 atm pressure and 300K while tank II is empty (P = 0). The valve is opened slowly so that gas can flow from tank I into tank II. The valve is closed once the pressure in tank I has reached 5 atm.

a) What is the temperature of the gas in tank I when the pressure there is 5 atm?
b) What is the temperature of the gas in tank II when the pressure in tank I has reached 5 atm?
c) Prove that this process is irreversible by calculating the total entropy change of the system.

**DATA:**

\[ C_p \text{ for the ideal gas is } \frac{7}{2} R \]
\[ C_v \text{ for the ideal gas is } \frac{5}{2} R \]

Note that pressure changes in the tank may be considered to be reversible processes. The expansion across the Joule-Thompson valve is irreversible but conserves enthalpy, as discussed in class. The complete enclosure of the tank is insulated so that no heat exchange with the environment occurs.

**Solution**

In all thermodynamic problems, it is useful to first identify the boundaries of the system and the resulting restrictions regarding the energy flows that can occur.

From the problem statement, we already identify that the combines Tank I – Tank II system does not exchange energy with the environment, since the system is rigid (no PV work) and insulated Q. Therefore:

\[ \Delta U_{Total} = 0 \]

Now the tricky part:

A common source of confusion is the fact that the number of moles in both Tank I and Tank II are not constant. This means that, if we want to write the first law expression, considering the instantaneous amount of gas that is in each tank, the first law would be in a differential equation form, which might be somewhat complicated.

It is much easier, however, to consider the system as follows:

- System 1: \( n_I \rightarrow \) Number of moles in the first tank, as soon as the pressure reaches 5 atm.
- System 2: \( n_{II} \rightarrow \) Number of moles that end up in the second tank, once the valve is closed.

Once we have defined the system is such a way, it is much easier to find the solution to the problem, as we shall see below:

**Part a)**

We are told that the Joule-Thompson valve opens very slowly. Additionally, there is no heat exchange with the environment. Therefore, we can use the expression for an adiabatic, reversible
For part b, since energy is conserved in the system, any change in energy in system $n_I$ has to be related to any change in energy in system $n_{II}$ through:

$$\Delta U_{TOT} = \Delta U_I + \Delta U_{II} = 0$$

$$\Delta U_I = -\Delta U_{II}$$

We know that a change in the internal energy for an ideal gas is only a function of its temperature change, and therefore:

$$\Delta U_I = n_I c_V \cdot (T_{I,f} - T_{I,0})$$

$$\Delta U_{II} = n_{II} c_V \cdot (T_{II,f} - T_{II,0})$$

We can then write

$$n_I \cdot (T_{I,f} - T_{I,0}) = n_{II} \cdot (T_{II,0} - T_{II,f})$$

This equation has two unknowns: one is $n_{II}$ and the other is $T_{II,f}$. To calculate $n_{II}$ we have to note that the total number of moles is conserved and that is equal to the initial amount of mass in Tank $I$, before the valve was opened.

$$n_{Tot} = n_I + n_{II} = \frac{P_{I,0} \cdot V}{R \cdot T_{I,0}}$$

$$n_{II} = \frac{P_{I,0} \cdot V}{R \cdot T_{I,0}} - \frac{P_{I,f} \cdot V}{R \cdot T_{I,f}}$$

From this and the previous equation, we obtain $T_{II,f}$. Note that $T_{II,f} > T_{II,0}$.

For part c, to prove that the whole process is irreversible, we have to prove that the entropy of the universe is greater than zero, i.e. $\Delta S > 0$:

$$\Delta S_{Universe} = \Delta S_I + \Delta S_{II} + \Delta S_{Environment} > 0$$

Since the total system does not exchange heat with the environment, $\Delta S_{Environment} = 0$. Moreover, since the system $n_I$ undergoes an adiabatic, reversible expansion, $\Delta S_I = 0$. Thus, we just have to calculate $\Delta S_{II}$.
Using the generalized expression for the change in entropy for an ideal gas,

$$\Delta S_{II} = n_{II} \left[ c_P \ln \left( \frac{T_{II,f}}{T_{II,0}} \right) - R \ln \left( \frac{P_{II,f}}{P_{II,0}} \right) \right]$$

And since, $P_{II,f} = 5 \text{ atm.}$, $P_{II,0} = 10 \text{ atm.}$ and $T_{II,f} > T_{II,0}$, $\Delta S_{II} > 0$ and

$$\Delta S_{Universe} > 0$$

The whole system undergoes an irreversible process. The source of irreversibility is the Joule-Thompson valve.