Course 18.327 and 1.130
Wavelets and Filter Banks

Matlab wavelet toolbox.
Matlab Example 3

1. 1-D signal analysis
Daubechies 9/7 pair: zeros of $H_0$ and $F_0$

```matlab
>> example3
Zeros of $H_0(z)$
ans =
    2.0311 + 1.7390i
    2.0311 - 1.7390i
   -1.0001 + 0.0001i
   -1.0001 - 0.0001i
   -0.9999 + 0.0001i
   -0.9999 - 0.0001i
    0.2841 + 0.2432i
    0.2841 - 0.2432i

Zeros of $F_0(z)$
ans =
    3.0407
   -1.0010 + 0.0010i
   -1.0010 - 0.0010i
   -0.9990 + 0.0010i
   -0.9990 - 0.0010i
    0.3289
```
Complete Set of Daub 9/7 Filters
Frequency Responses of Daub 9/7

Frequency responses of Daubechies 9/7 filters

Angular frequency (normalized by \( \pi \))

Frequency response magnitude
Single Stage Decomposition

Single stage wavelet decomposition

- $x$
- $y_0$
- $y_1$
- $x\hat{}$
3-Stage Decomposition

Tree Decomposition

Data for node: (7) or (3,0)
3-Stage Decomposition

Three stage wavelet decomposition
Individually Reconstructed Branches
Reconstruction Error --- upcoef
Reconstruction Error --- wrcoef
Reconstruction Error --- waverec
Matlab Example 4

1. 2-D image analysis
Original Image
Wavelet Decomposition

Tree Decomposition

data for node: (1) or (1,0).
Single Stage Decomposition
Two-Stage Decomposition
Reconstructed Branches

Original

ra1  rh1  rv1  rd1

ra2  rh2  rv2  rd2
Global Hard Thresholding

Original

Compressed using global hard threshold

Energy retained = 99.8%
Null coefficients = 74.1%
Variable Hard Thresholding

Original

Compressed using variable hard thresholds

Energy retained = 99.8%
Null coefficients = 74.3%