1. (a) Write a function with “signature”

\[
\text{function } [\text{bern_rvs}] = \text{Bernoulli}(n,\theta)
\]

which returns a row vector \( \text{bern_rvs} \) of \( n \) independent random variables (more precisely, realizations of independent random variables) drawn from the Bernoulli probability mass function \( f_X(x;\theta) \) for given \( \theta \equiv \theta \). Note the inputs \( n \) and \( \theta \) are scalars. Your function should take advantage of the MATLAB built-in \( \text{rand} \) — called as \( \text{rand}(1,n) \) to create a row vector.

(b) Then write a script which calls your function \( \text{Bernoulli} \) for \( n = 1000 \) and \( \theta = 0.25 \) and furthermore calculates and displays

\[
\text{frac_one} = \frac{1}{n} \sum_{i=1}^{n} \text{bern_rvs}(i),
\]

which is simply the fraction of “one” entries in your random vector (realization). Of course \( \text{frac_one} \) should be roughly \( \theta \). Make sure to run your script for several different sets of inputs \((n,\theta)\) to \( \text{Bernoulli} \) in order to confirm that both the script and \( \text{Bernoulli} \) are working correctly.

2. (a) Write a function with “signature”

\[
\text{function } [\text{x1pts},\text{x2pts}] = \text{unif_over_rect}(a1,b1,a2,b2,n)
\]

which provides the coordinates \((\text{x1pts}(i),\text{x2pts}(i)), 1 \leq i \leq n\) of \( n \) random darts (more precisely, realizations of random darts) thrown at the rectangle \( a1 \leq x_1 \leq b1, a2 \leq x_2 \leq b2 \). (The lower left corner of the rectangle is \( a1,a2 \); the upper right corner of the rectangle is \( b1,b2 \).) You may assume that the darts are drawn from the bivariate uniform distribution over the rectangle and hence correspond to independent random variables \( \text{x1pts}(i) \) and \( \text{x2pts}(i) \).

Note that \( \text{x1pts} \) and \( \text{x2pts} \) should each be single-index row arrays of \( \text{length } n \) — two \textit{separate} outputs. (In the next recitation we will consider double-index arrays.) The inputs \( a1, b1, a2, b2 \), and \( n \) are all scalars.
(b) Then write a script which calls `unif_over_rect` for $a_1 = -1, b_1 = 1, a_2 = -1, b_2 = 1, n = 2000$ and calculates and displays $\text{frac\_in\_circ}$, the fraction of darts that fall inside the unit circle (radius unity) centered at the origin. Of course $\text{frac\_in\_circ}$ should be close to $\pi/4$. 