LAST TIME

\[ \rightarrow \bigcirc \] absorption, scattering, extinction \} cross-section efficiency

- ALBEDO
- PHASE FUNCTION

MIE SCATTERING
... FOR SPHERICAL PARTICLE

size parameter \( X = \frac{2\pi r}{\lambda_0} \rvert, \quad M = \frac{N_I}{N} \)

\[ \ldots \text{SPECIAL SOLUTIONS} \]

RAYLEIGH SCATTERING \( X \ll 1 \implies Q_s \sim X^4 \)

GEOMETRIC OPTICS \( X \gg 1 \)

(RAY TRACING)

\[ Q_a \sim X \]

(at nanoscale) absorption is more important

GROUP OF IDENTICAL PARTICLES

\[ N_I = \text{PARTICLES PER UNIT VOL.} \]

\[ \begin{align*}
\text{SCATTERING COEFFICIENT: } & \quad \sigma_{s\lambda} = N_I C_3 \left[ \frac{1}{m} \right] \\
\text{Absorption Coeff: } & \quad K_\lambda = N_I C_a \\
\text{Extinction Coeff: } & \quad \beta_\lambda = K_\lambda + \sigma_{s\lambda}
\end{align*} \]
**Particle Distribution Function**

\[ n(r) = \left[ \frac{1}{m^3} \cdot \frac{1}{m} \right] \]

\[ \sigma_{\lambda} = \int_0^{\infty} n(r) c_s \, dr \quad \text{"A weighted average"} \]

**Volume Fraction**: \[ f_0 = \frac{1}{3} \pi r^3 n(r) \, dr \]

**Phase Function**:

\[ \Phi(\omega' \rightarrow \omega) = \frac{1}{\sigma_{\lambda}} \int_0^{\infty} c_s \phi(\omega' \rightarrow \omega, r) \, n(r) \, dr \]

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**Independent vs. Dependent Scattering**

- **Particles can scatter waves** from to **neighboring particles**
- **The closer particles are**, the stronger they interfere **with one another**

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\[ \text{INDEPENDENT REGION} \]

\[ \text{DEPENDENT REGION} \]

\[ f_0 \quad (\text{VOLUME FRACTION}) \]
SPECTRAL PROPERTIES OF MATERIALS

ENERGY STATES IN A MATERIAL GOVERNED BY QUANTUM MECHANICS

ENERGY CONSERVATION

\[ E_{\text{final}} = E_{\text{initial}} + \hbar \omega \]

OR

\[ E_f - E_i = \hbar \omega_p \]

MOMENTUM CONSERVATION

\[ \mathbf{t}_{ik} = \mathbf{P}_f - \mathbf{P}_i + \mathbf{G} \]

MATERIAL ENERGY STATES

... A SINGLE ATOM

\[ \text{Nucleus, } \text{e}^- \]

TRANSLATIONAL ENERGY OF THE ATOM

\[ E = \frac{m v^2}{2} \]

de BROGLIE WAVE

\[ \lambda = \frac{\hbar}{p} \]

HYDROGEN ATOM:

\[ E_n = -\frac{13.6 \text{ eV}}{n^2}, \quad n = 1, 2, 3, \ldots \]

\( n \) = PRINCIPLE QUANTUM NUMBER

\[ \hbar \omega_{\text{photon}} = -13.6 \text{ eV} \left( \frac{1}{2^2} - 1 \right) = \frac{3}{4} \times 13.6 \text{ eV} \]
NEED PRINCIPLE QUANTUM #S FOR EACH DIMENSION (D.O.F.)

Thus, for 3-D our wave function takes the form

\[ \psi_{n\ell m} \text{ (each } n, \ell, m, s \text{ combo defines a quantum state)} \]

Constraints:
\[ \ell < n \]
\[ |m| \leq \ell \]

Electrons
\[ \psi_{100s} \Rightarrow \psi_{100(\frac{1}{2})}, \psi_{100(-\frac{1}{2})} \]
\[ s = \pm \frac{1}{2} \]

\[ h = 1 \text{ has 2 states} \]
\[ h = 2 \text{ has 8 states} \]

**Lithium, not so stable**

**Diatomic molecules**

Can have rotation
\[ E_\ell = \frac{\hbar^2}{2I} \ell(\ell+1) \]
\[ I \text{ = moment of inertia } m r^2 \]

\[ E_{\ell+1} - E_\ell = \hbar \omega_p \]
VIBRATION

\[ E_n = \hbar \omega (n + \frac{1}{2}) \]
\[ \omega = \frac{k}{\sqrt{m_{\text{eff}}}} \]

SOLIDS

1-D HYDROGEN

\[ k = a \rightarrow \]

\[ n = 2 \]

\[ n = 1 \]

\[ \text{WE USE } k \text{ RATHER THAN } n \]

\[ \text{SPLITTING DUE TO 2 ATOMS} \]
\[ \text{\&\text{\&}} \text{ PAULI EXCLUSION} \]

\[ \text{BANDS DUE TO SEVERAL ATOMS} \]

\[ k_i = \frac{2\pi i}{L} = \frac{2\pi i}{N_a} \]