Gear geometry

Consider the curve generated by unwrapping a string from around a disk of radius \( R_B \). The end of the string will trace an involute curve.

To mathematically define an involute consider the following:

- \( R_c = \text{length\_of\_string\_unwrapped} \)
- \( \tan(\phi) = \frac{R_c}{R_B} \)
- \( R_B = \text{radius\_of\_generating\_cylinder} \)
- \( \phi = \text{pressure\_angle} \)
- \( \theta = \text{position\_parameter\_associate\_with\_involute} \)
- \( E = \theta + \phi \)
- point at loose end of curve is at polar coordinates \( R, \theta \)
- \( E = \text{interim\_variable\_sum\_of\_angles} \)
- length of arc = radius * angle

\[
R_c = \frac{R_B \cos(\phi)}{\cos(\phi)} = \frac{R_B}{\cos(\phi)}
\]

\[
\theta = \tan(\phi) - \phi
\]

basic definition for angular coordinate of involute curve for any \( \phi \). Curve is generated by setting \( \phi \) to range from 0 to max

from geometry ...

\[
\cos(\phi) = \frac{R_B}{R} \Rightarrow R = \frac{R_B}{\cos(\phi)}
\]

the other coordinate, \( R = \text{pitch\_radius} \) when \( \phi = \text{pressure\_angle\_for\_design} \)

\[
\begin{align*}
\phi &= 40\text{deg} & \text{pressure\_angle} \\
\theta_1 &= 0, 0.01.. 2\pi & \text{2\_\pi\_range\_variable} \\
\theta &= \tan(\phi) - \phi & \text{involute}(\phi) \\
\phi_1 &= 0.877\text{deg} & \text{R\_rad} = 0, 0.1.. 2 \\
R_B &= 1 & \text{in this case we will define the base radius} \\
& \\
\text{calculate the pitch radius} & \quad R_p = \frac{R_B}{\cos(\phi)} & \quad R_p = 1.305 \\
& \quad N.B. \text{ positive directions for } \theta \\
& \quad \text{and } \phi \text{ are opposite} \\
& \quad \text{the involute is constructed by varying a dummy pressure angle over a range - equivalent to unwrapping the string from the disk.} \\
& \quad \phi_1 = 0, 0.01.. \phi_1\_\text{max} & \text{range\_variable\_for\_construction} \\
& \quad \theta_2(\phi_1) = \tan(\phi_1) - \phi_1 & \quad R_2(\phi_1) = \frac{R_B}{\cos(\phi_1)} \\
& \quad \text{a tangent is drawn from the pressure angle thru the involute at the pitch radius (perpendicular to involute)} \\
& \quad R_{\text{tan}} := \begin{cases} 
R_p & \frac{\pi}{2} \\
R_B & \frac{\pi}{2} - \phi 
\end{cases} & \text{draws the tangent} \\
& \quad R_{\text{tan}} = \begin{pmatrix}
1.305 & 1.571 \\
1 & 0.873
\end{pmatrix}
\]
add in an involute at a nominal pressure angle of 50 deg and then rotate it by the difference between pressure angles. Notice it overlays the first tangent.

\[
\phi_4 := 50^\circ \quad \theta_4 := \tan(\phi_4) - \phi_4 \quad \theta_4 = 18.282^\circ \quad (\phi_4 - \phi) \cdot k_4 \quad \text{does the rotation with } k_4 = 1
\]

\[
R_{tan1} := \begin{bmatrix}
\frac{R_B}{\cos(\phi_4)} & \frac{\pi}{2} + (\phi_4 - \phi) \cdot k_4 \\
R_B & \frac{\pi}{2} - \phi_4 + (\phi_4 - \phi) \cdot k_4
\end{bmatrix}
\]

\[
R_{tan1} = \begin{bmatrix} 1.556 & 1.745 \\ 1 & 0.873 \end{bmatrix}
\]

the resulting figure is as follows:

**tooth construction (design)**

at this point we know ...

\[
R_B = \text{radius\_of\_generating\_cylinder}
\]

\[
\phi = \text{pressure\_angle}
\]

\[
R = \frac{R_B}{\cos(\phi)} \quad \text{radius as function of pressure angle} = \text{pitch radius at design pressure angle}
\]

define

\[
CP = \text{circular\_pitch} = \frac{\text{circumference\_of\_pitch\_diameter}}{\text{number\_of\_teeth}}
\]
set pressure angle \( \phi := 25 \text{deg} \)

\[
\text{DP} := 10 \quad \text{diametral\_pitch} = \frac{\text{DP}}{\text{pitch\_diameter}} \quad \frac{N_G}{2R_G} = \frac{N_P}{2R_P} \quad \text{CP\_DP} = \pi
\]

\( N_P := 20 \quad \text{number\_of\_pinion\_teeth} \quad N_G := 30 \quad \text{number\_of\_gear\_teeth} \)

\( BL := 0.01 \quad \text{backlash} = 0.01 \quad \text{beyond scope, depends on DP} \)

\( \text{CTT}_P := \frac{\pi}{DP} - \frac{BL}{2} \quad \text{circular\_tooth\_thickness} \)

\( \text{calculate pitch and base radii} \quad \text{CTT}_G := \text{CTT}_P \quad \text{same on pitch diameter} \)

\[
R_G := \frac{N_G}{DP} \quad R_G = 1.5 \quad \text{pitch\_radius\_gear} \quad R_{BG} := R_G \cdot \cos(\phi) \quad R_{BG} = 1.359 \quad \text{base\_diameter\_gear}\]

\[
R_P := \frac{N_P}{DP} \quad R_P = 1 \quad \text{pitch\_radius\_pinion} \quad R_{BP} := R_P \cdot \cos(\phi) \quad R_{BP} = 0.906 \quad \text{base\_diameter\_pinion}\]

\[
C := R_G + R_P \quad C = 2.5 \quad \text{center\_distance}\]

\[
R := \frac{R_G}{R_P} \quad R = 1.5 \quad \text{gear\_ratio} \quad \text{i.e. gear\_ration\ is\ ratio\ of\ pitch\ radii\ (or\ diameters\ or\ number\ of\ teeth)}
\]

\[
\text{CTT}_{P2} = 2R_P \left( \frac{\text{CTT}_P}{2R_P} + \text{inv}(\phi_1) - \text{inv}(\phi_2) \right) \quad \text{derived\ from\ involute\ geometry}
\]

\[
\text{at } R_2 \text{ point on thickness of tooth } B \text{ is} \quad B = \theta_1 + \frac{1}{2} \frac{\text{CTT}_1}{R_1} - \theta_2 \quad \text{inv}(\phi) := \tan(\phi) - \phi
\]

\( \text{derived\ below} \ldots \)
\[ A = \theta_1 + \frac{1}{2} \frac{\text{CTT}_1}{R_1} \]

\[ \text{CTT}_1 = \text{circular_tooth_thickness} \]
\[ \phi = \text{pressure_angle_design} \]
\[ \theta_1 = \text{involute_of_design_pressure_angle} \]
\[ R_1 = \text{pitch_radius} = \frac{R_B}{\cos(\phi)} \]

Here consider varying \( \phi \) from 0 to a value > design angle = \( \phi_2 \)

\[ \theta_2 = \text{involute_of}_\phi_2 \]
\[ B(\phi_2) = A - \theta_2 \]
\[ R_2 = \frac{R_B}{\cos(\phi_2)} \]

so ..
\[ B = \theta_1 + \frac{1}{2} \frac{\text{CTT}_1}{R_1} - \theta_2 \]

and points on tooth surface are \( R_2, B \)

additional definitions

addendum dedendum root_diameter

tooth profile ... with pitch radius and base radius shown ...

plot set up
move the pinion out to C, rotating it by $\pi$ and offsetting both by half tooth thickness $\theta_{plot_G}(R_G)$

geometry to shift circle
plot set up