Chapter 5
Kinetics
Fission chain reaction

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Delayed neutrons

Delayed neutrons emitted from the decay of fission products long after the fission event. Delay is caused by half-life of beta decay of delayed neutron pre-cursor nucleus.
Delayed neutrons

Delayed neutrons are grouped into 6 groups of delayed neutron precursors with an average decay constant $\lambda_i$ defined for each.

$\lambda_i \equiv$ The decay constant for the $i^{th}$ group of delayed neutrons

$\beta_i \equiv$ The fraction of fission neutrons emitted by delayed neutron precursor group $i$.

$\beta \equiv$ The fraction of fission neutrons that are delayed.

$$\beta \approx 0.0075 \quad \sum_{i=1}^{6} \beta_i$$
Point Kinetic Equations

For an initially critical system

\[ \frac{dp(t)}{dt} = \left( \frac{\rho(t) - \beta}{\Lambda} \right) p(t) + \sum_{k=1}^{K} \lambda_k c_k(t) \]

\[ \frac{dc_k(t)}{dt} = \frac{\beta_k}{\Lambda} p(t) - \lambda_k c_k(t) \]

Most important assumption: Assumes that the perturbation introduced in the reactor affects only the amplitude of the flux and not its shape.
Dynamic Reactivity $\rho(t)$

- Most important kinetics parameter
  - Its variations are usually the source of changes in neutronic power
  - Only term that contains the neutron loss operator (M operator)
    - Associated to control mechanisms
    - Also sensitive to temperature
  - No units
    - Expressed in terms of mk (milli-k) or pcm
Delayed-Neutron Fraction $\beta(t)$

- Effective delayed neutron fraction is linked to the constants of each fissionable isotope which measure the fraction of fission product precursors
  - Called “effective” because it is weighted by the flux in the reactor
- Can vary with burnup
  - Different values exist at BOL and EOL
  - Variations in burnup are on a much larger time scale than usual range of application of point kinetics equations
Prompt-Neutron Lifetime

• Measure of the average time a neutron survives after it appears as either a prompt neutron or a delayed-neutron
Solution with one effective delayed neutron precursor group

\[ \frac{dP(t)}{dt} = \left( \frac{\rho_0 - \beta}{\Lambda} \right) P(t) + \lambda C(t) \]

\[ \frac{dC(t)}{dt} = \frac{\beta}{\Lambda} P(t) - \lambda C(t), \quad t \geq 0 \]

Step reactivity change

\[ t < 0 \Rightarrow \rho(t) = 0, \quad P(t) = P_0 \]

\[ t \geq 0 \Rightarrow \rho(t) = \rho_0 \]
Solution

\[ P(0) = P_0, \quad C(0) = \frac{\beta}{\lambda \Lambda} P_0 \]

Initial conditions

\[ P(t) = Pe^{st}, \quad C(t) = Ce^{st} \]

Solutions

\[ sP = \left[ \frac{\rho_0 - \beta}{\Lambda} \right] P + \lambda C \]

Substitute

\[ sC = \frac{\beta}{\Lambda} P - \lambda C \]
Solution

\[
\begin{bmatrix}
\rho_0 - \beta \\
-\frac{\beta}{\Lambda} & s + \lambda
\end{bmatrix}
\begin{bmatrix}
P \\
C
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix}
\]

Linear Homogeneous System

Non-trivial solution if and only if \( \det A = 0 \)

\[
\Lambda s^2 + (\lambda \Lambda + \beta - \rho_0) s - \rho_0 \lambda = 0
\]
Solution

\[ s_{1,2} = \frac{1}{2\Lambda} \left[ -\left( \beta - \rho_0 + \lambda \Lambda \right) \pm \sqrt{\left( \beta - \rho_0 + \lambda \Lambda \right)^2 + 4\lambda \Lambda \rho_0} \right] \]

\[ P(t) = P_1 e^{s_{1t}} + P_2 e^{s_{2t}} \quad \text{and} \quad C(t) = C_1 e^{s_{1t}} + C_2 e^{s_{2t}} \]
Approximate solution

Assume \( \frac{\lambda \Lambda}{\beta} \ll 1 \) and \( |\rho_0| \ll \beta \)

\[ s_1 \equiv \frac{\lambda \rho_0}{\beta - \rho_0}, \quad s_2 \equiv -\left( \frac{\beta - \rho_0}{\Lambda} \right) \]

\[
P(t) \equiv P_0 \left[ \left( \frac{\beta}{\beta - \rho_0} \right) \exp \left( \frac{\lambda \rho_0}{\beta - \rho_0} \right) t - \left( \frac{\rho_0}{\beta - \rho_0} \right) \exp \left( \frac{\beta - \rho_0}{\Lambda} \right) t \right]
\]
Solution

This solution is not valid for large changes in reactivity!

\[ P(t) \equiv P_0 \left[ \left( \frac{\beta}{\beta - \rho_0} \right)^\lambda \exp \left( \frac{\lambda \rho_0}{\beta - \rho_0} t \right) - \left( \frac{\rho_0}{\beta - \rho_0} \right)^\lambda \exp \left( \frac{\rho_0 - \beta}{\Lambda} t \right) \right] \]

\[ |\rho_0| \quad 0.0025, \ \beta \quad 0.0075, \ \lambda \quad 0.08\text{s}^{-1}, \ \Lambda \quad 10^{-3}\text{s} \]
Reactor Period

- Defined as the power level divided by the rate change of power

\[ \tau(t) = \frac{p(t)}{\frac{dp(t)}{dt}} = \frac{p(t)}{\tau(t) \frac{dp(t)}{dt}} \]

- Period of infinity implies steady-state
- Small positive period means a rapid increase in power
- Small negative period means rapid decrease in power
- If period is constant, power varies according to

\[ p(t) = p_0 e^{t/\tau} \]
Reactor Period

• For the case with one delayed group, the reactor period can be separated in two parts
  – Prompt period
  – Stable period

• The solution has two exponential and they usually have very different coefficients.
Scenario 1

• $\rho_0 < 0$
  – Corresponds to a quick reactor shutdown
  – Both roots are negative
  – $s_2 << s_1$ thus the power drops almost instantly to a fraction of its initial power (prompt drop)
  – However, it is impossible to stop a reactor instantaneously
Example

• The second root is so small that in the matter of a fraction of second becomes inconsequential

• Power drops almost instantly to the coefficient of the first exponential term

\[ P(t) \cong P_0 \left[ \left( \frac{\beta}{\beta - \rho_0} \right) \exp\left( \frac{\lambda \rho_0}{\beta - \rho_0} \right) t - \left( \frac{\rho_0}{\beta - \rho_0} \right) \exp\left( \frac{\rho_0 - \beta}{\Lambda} \right) t \right] \]

• Thus the stable period is equal to 1/s₁

• And the prompt period is equal to 1/s₂

Courtesy of canteach@candu.org. Used with permission.
Demo

• Negative Step insertion of -12mk
• Parameters
  – Beta = 0.006
  – LAMBDA = 0.001
  – Lambda = 0.1 s\(^{-1}\)
• Power drops by 33% almost instantly, and then decays slowly
Scenario 2

- $0 < \rho_0 < \beta$
- One root is positive and the other is negative
- Power increases rapidly and the grows exponentially
• Power increases rapidly by $\beta/(\beta - \rho)$
  – Positive prompt jump
• Stable period is equal to $1/s_1$
• Prompt period is equal to $1/s_2$
Demo

• Positive Step insertion of 1mk
• Parameters
  – Beta = 0.006
  – LAMBDA = 0.001
  – Lambda = 0.1 s\textsuperscript{-1}
• If rho approaches beta, the stable period becomes very short
Scenario 3

- $\rho_0 > \beta$ (prompt super-critical)
- Reactor is critical without the need of the delayed neutrons
- One root is positive and one is negative
- Reactor period becomes less than 1s
• Power increases at a very rapid rate

• Disastrous consequences
  – Unless a feedback mechanism can cancel out the reactivity
Demo

- Positive Step insertion of 7mk
- Parameters
  - Beta = 0.006
  - LAMBDA = 0.001
  - Lambda = 0.1 s\(^{-1}\)
- Reactor is critical (or supercritical) without the presence of delayed neutrons
  - Prompt jump dominates
Limiting cases- Small reactivity insertions

\[ \rho_0 \ll \beta, \text{ thus } |s_1| \ll \lambda_1 < \lambda_2 \ldots < l^{-1} \]

\[ T = \frac{1}{s_1} = \frac{1}{\rho_0} \left[ l + \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i} \right] \equiv \frac{\langle l \rangle}{\rho_0} \equiv \frac{\langle l \rangle}{k - 1} \]

\[ \rho_0 \gg \beta, \text{ thus } s_1 \gg \lambda_i \]
Limiting cases—Large reactivity insertions

\[ \rho_0 \equiv \frac{s_1}{s_1 + l^{-1}} + \frac{l^{-1}}{s_1 + l^{-1}} \sum_{i=1}^{6} \beta_i = \frac{s_1 + \beta l^{-1}}{s_1 + l^{-1}} \]

\[ T = \frac{1}{s_1} \equiv \frac{l}{k(\rho_0 - \beta)} \equiv \frac{l}{k - 1} \]
Positive Reactivity

Graphical Solution of Inhour Equation: Positive Reactivity
Negative Reactivity

Graphical Solution of Inhour Equation: Negative Reactivity
## Typical parameters

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<th>LWR</th>
<th>CANDU</th>
<th>Fast Reactor</th>
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<td>$1 \times 10^{-3}$</td>
<td>$1 \times 10^{-6}$</td>
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<td>$\lambda$</td>
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