Operational Reactor Safety
22.091/22.903

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Professor of the Practice

Lecture 4
Fuel Depletion & Related Effects
Topics to Be Covered

• Fuel “burnup”
• Transmutation
• Conversion/Breeding
• Samarium 149
• Xenon 135
• Operational Impacts
Fuel Burnup

• Depletion Equation

• Definition of burnup
  – thermal energy output per mass of fuel
  – MWD/MTHM
Transmutation

• Equation for production of any nuclide

• Conversion versus Breeding
  – Depending on core physics design of the reactor core
  – $\eta$ (eta)
    • Number of neutrons produced/absorbed in fuel

• Conversion ratio
  – rate of creation of new fissile/destruction of existing fissile
FIGURE 6-1
Values of eta [$\eta$] for fissile nuclides as a function of energy. [Courtesy of Electric Power Research Institute (Shapiro, 1977).]
# Breeding Ratios for Reactor Systems

## TABLE 6-1
Average Conversion or Breeding Ratios for Reference Reactor Systems

<table>
<thead>
<tr>
<th>Reference reactor</th>
<th>Initial fuel $^\dagger$</th>
<th>Conversion cycle $^\dagger$</th>
<th>Conversion ratio</th>
<th>Breeding ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>BWR</td>
<td>2–4 wt% $^{235}\text{U}$</td>
<td>$^{238}\text{U}\text{–Pu}$</td>
<td>0.6</td>
<td>—</td>
</tr>
<tr>
<td>PWR</td>
<td>2–4 wt% $^{235}\text{U}$</td>
<td>$^{238}\text{U}\text{–Pu}$</td>
<td>0.6</td>
<td>—</td>
</tr>
<tr>
<td>PTGR</td>
<td>1.8–2.1 wt% $^{235}\text{U}$</td>
<td>$^{238}\text{U}\text{–Pu}$</td>
<td>$\geq 0.6$</td>
<td>—</td>
</tr>
<tr>
<td>PHWR</td>
<td>Natural U</td>
<td>$^{238}\text{U}\text{–Pu}$</td>
<td>0.8</td>
<td>—</td>
</tr>
<tr>
<td>HTGR</td>
<td>$\approx 5$ wt% $^{235}\text{U}$</td>
<td>$^{232}\text{Th}\text{–}^{233}\text{U}$</td>
<td>0.8</td>
<td>—</td>
</tr>
<tr>
<td>LMFBR</td>
<td>10–20 wt% Pu</td>
<td>$^{238}\text{U}\text{–Pu}$</td>
<td>—</td>
<td>1.0–1.6</td>
</tr>
</tbody>
</table>

$^\dagger$ All plutonium in power reactors is an isotopic mixture based on initial conversion of $^{238}\text{U}$ to $^{239}\text{Pu}$ and followed by transmutation to the “higher” isotopes.
Buildup of Plutonium with Burnup

**FIGURE 6-2**
Buildup of plutonium isotopes with burnup for a representative LWR fuel composition.

http://atom.kaeri.re.kr/
### TABLE 6-2
Reactivity Penalty from Selected Transmutation Products for Recycle of BWR Fuel†

<table>
<thead>
<tr>
<th>End of cycle number</th>
<th>Reactivity penalty at discharge, %Δk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$^{236}$U ‡</td>
</tr>
<tr>
<td>1</td>
<td>0.62</td>
</tr>
<tr>
<td>2</td>
<td>0.90</td>
</tr>
<tr>
<td>3</td>
<td>1.12</td>
</tr>
</tbody>
</table>

‡The $^{236}$U concentration is assumed not to decrease in the diffusion plant.
§Neptunium and americium are removed by reprocessing on each recycle.
Fission Products

• Fission Fragments ➔ Fission Products
  – Rate of Creation - \( \gamma \Sigma_f \Phi \)
  – \( \gamma \) fission yield

• Fission Fragment Balance Equation
Samarium Buildup

Basic Theory

\[ \begin{align*}
\frac{^{149}\text{Nd}}{\beta^- 1.7 \text{ h}} & \quad \frac{^{149}\text{Pm}}{\beta^- 53 \text{ h}} \quad \frac{(n, \gamma) 40. \times 10^3 \text{ b}}{^{150}\text{Sm}} \\
\end{align*} \]

\[ \begin{align*}
\text{FISSILE NUCLIDE} & \quad \gamma^{\text{Nd}} \\
^{233}\text{U} & \quad 0.0066 \\
^{235}\text{U} & \quad 0.0113 \\
^{239}\text{Pu} & \quad 0.0190
\end{align*} \]
FIGURE 6-6
Behavior of $^{149}\text{Sm}$ in representative LWR fuel: (a) decay and reaction chain, (b) fission yields, (c) concentration vs. time.
Xenon Buildup

### Basic Theory

![Diagram of Xenon Buildup]

<table>
<thead>
<tr>
<th>FISSILE NUCLIDE</th>
<th>$\gamma^{135\text{Te}}$</th>
<th>$\gamma^{135\text{Xe}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{233}\text{U}$</td>
<td>0.051</td>
<td></td>
</tr>
<tr>
<td>$^{235}\text{U}$</td>
<td>0.061</td>
<td>0.003</td>
</tr>
<tr>
<td>$^{239}\text{Pu}$</td>
<td>0.055</td>
<td></td>
</tr>
</tbody>
</table>

\[ (b) \]
\[ 3 \times 10^{15} \]

\[ 2 \times 10^{15} \]

\[ 1 \times 10^{15} \]

\[ 0 \]

\[ \frac{^{135}Xe \text{ CONCENTRATION at/cm}^3}{0 \quad 20 \quad 40 \quad 0 \quad 20 \quad 40 \quad 60 \quad 80 \quad 100} \]

\[ \text{INITIAL STARTUP} \quad \text{SHUTDOWN} \]

\[ \text{RESTART} \]

\[ \text{TIME FROM INITIAL STARTUP, h} \]

**FIGURE 6-7**

Behavior of \(^{135}\text{Xe}\) in representative LWR fuel: (a) decay and reaction chain, (b) fission yields, (c) concentration vs. time.
**FIGURE 6-8**  
Poisoning of $^{135}$Xe as a function of time after shutdown for a representative LWR fuel composition at various neutron flux levels. Curve 1: $\Phi = 1 \times 10^{13}$ n/cm$^2$·s; Curve 2: $\Phi = 5 \times 10^{13}$ n/cm$^2$·s; Curve 3: $\Phi = 1 \times 10^{14}$ n/cm$^2$·s; Curve 4: $\Phi = 5 \times 10^{14}$ n/cm$^2$·s.
Operational Impacts

- Xenon Oscillations
- Fuel Design for cycle length of core
- Fuel management strategies
- Power peaking limits
- Power distribution control
Reactor Physics Calculations

- Multi-Group Diffusion Equations
  - Model core – using fuel pin and assembly homogenization of materials and fuels with pins averaged horizontally but detailed axially
- Run Static calculation for core power and flux distribution
- Fluxes used to perform depletion calculations as noted for a “time step”
- New material calculations used to produce new power and flux distribution for next “time step” – 1 month
- Incorporate only significant isotopes – high absorption and/or fission cross sections ignoring short lived isotopes in decay chains. – use lumping procedure
- Need to consider early xenon and Samarium build up 50 hours/500 hours
- Track key isotopes for all fuel assemblies for refueling management
Homework Assignment

- Chapter 6
  - Problems: 6.2, 6, 7, 9, 11, 15