Electricity and Magnetism

• Electric field continued
Electric Field

- New concept – $\vec{E}$
- Charge $Q$ gives rise to a Vector Field

$$\vec{E}(\vec{x}) = \frac{\vec{F}(\vec{x})}{q}$$

- $\vec{E}$ is defined by strength and direction of force on small test charge $q$
Electric Field

• For a single charge

\[ E = k \frac{Q}{r^2} \]

• Visualize using Field Lines
  - Cartoon!
  - Strength -> Density of Lines
  - Direction -> Direction of Lines
    • away from positive charges
Electric Field

• Field can be used to accelerate charged particles

\[ F = Q E \]

-> Particle Accelerators
The Electric Field

• Electric Field also exists if test charge q is not present

• We can say:

The charge Q gives rise to a property of space itself – the Electric Field

-> In-Class Demo...
Electric Field Demo

- Use a Van-der-Graaf Generator
- Much more powerful than rubbing glass rods
- Not really dangerous (I’ve been told) – but potentially painful
- Creates large electric fields
- Really big ones were used in Particle Accelerators (still in use in some labs)
Torque $\tau = \vec{p} \times \vec{E}$

$\vec{p} = Q \vec{l}$ Dipole moment
Does Dipole feel a net Force?

No

Yes
Superposition Principle

- Field of many charges is Vector Sum of individual fields
Example: Superposition principle for 3 charges

\[ \vec{E}_i = k \frac{Q_i}{r_i^2} \hat{r}_i \]

\[ E_{y,\text{total}} = 0 \]

\[ E_{i,x} = E_i \cos(\Theta) = E_i \frac{x_0}{x_0^2 + y_i^2} \]

\[ E_x = \sum E_{i,x} \]

\[ = kQ \left( \frac{1}{x_0^2} + \frac{2x_0}{(x_0^2 + L^2)^{3/2}} \right) \]
Example: Superposition principle for continuous charge distribution

Total Charge $Q = 2 \lambda L$

$dQ = \lambda \ dy$

$d\vec{E} = k \frac{dQ}{r^2} \hat{r}$

\[ dE_x = k \frac{dQ}{x_0^2 + y^2} \cos(\Theta) \]
\[ = k \frac{dQ}{x_0^2 + y^2} \frac{x_0}{\sqrt{x_0^2 + y^2}} \]
\[ = k \lambda dy \frac{x_0}{(x_0^2 + y^2)^{3/2}} \]

Feb 15 2002
Example: Superposition principle for continuous charge distribution

\[ \vec{E} = E_x \hat{x} \]

\[ = \int_{all\,charge} dE_x \]

\[ = k \lambda x_0 \int_{-L}^{+L} \frac{dy}{(x_0^2 + y^2)^{3/2}} \]

\[ \vec{E}_x = 2k \lambda \frac{L}{x_0 \sqrt{x_0^2 + L^2}} \]

\[ d\vec{E} = k \frac{dQ}{r^2} \hat{r} \]

\[ dE_x = k \frac{dQ}{x_0^2 + y^2} \cos(\Theta) \]

\[ = k \frac{dQ}{x_0^2 + y^2} \frac{x_0}{\sqrt{x_0^2 + y^2}} \]

\[ = k \lambda dy \frac{x_0}{(x_0^2 + y^2)^{3/2}} \]
More on Fields and Field Lines

• What’s wrong with this picture?
• Magnitude and direction of field have to be unique at each point!
• Field lines can’t cross!
More on Fields and Field Lines

- Very close to surface of charged object
- Field lines perpendicular to surface (if we go close enough)!
- Symmetry left and right (like an infinite plane)