Electricity and Magnetism

• More on
  – Electric Flux
  – Gauss’ Law
More on Electric Flux and Gauss’ Law

\[ \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \]

\[ \oint \vec{B} \cdot d\vec{l} = 0 \]

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \]

\{ Maxwell Equations (1873) \}

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Electric Flux

\[ \Phi_E = \vec{E} \cdot \vec{A} \]

Note absence of ','

‘\( \Phi_E \)’ is a Scalar: How much?

I.e. how much field passes through surface \( A \)?
• **Direction**
  - Normal to surface

• **Magnitude**
  - Surface Area

• **For closed surface**
  - Pointing outwards
Electric Flux

- What if $\vec{E}$ not constant on surface $A$?
- Use integral

$$\Phi_E = \int_A \vec{E} \cdot d\vec{A}$$

- Often, ‘closed’ surfaces
Gauss’ Law

• Connects Flux through closed surface and charge inside this surface:

\[ \oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} \]

Note: \( k = \frac{1}{4\pi \epsilon_0} \)
Gauss’ Law

\[ \oint_A \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \]

- True for ANY closed surface around \( Q_{encl} \)
- Suitable choice of surface \( A \) can make integral very simple
Use the Symmetry!

Point Charge

\[ \vec{E} \parallel d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = E dA \]

\[ E(r) = \text{const.} \Rightarrow \int_A E \cdot dA = E \int_A dA = EA \]

Spherical Surface

(1) \( r > r_0 \):

\[ \int_{A_1} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{enc}}}{\varepsilon_0} \]

\[ \Rightarrow E = \frac{1}{4\pi \varepsilon_0 r^2} \]

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Use the Symmetry!

Charged Sphere

Spherical Surfaces

\[ \vec{E} \parallel d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = EdA \]

\[ E(r) = \text{const.} \]
Use the Symmetry!

(1) $r > r_0$:
\[
\int_{A_1} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{enc}}}{\epsilon_0}
\]
\[\implies E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2}\]

(2) $r < r_0$:
\[
\int_{A_2} \vec{E} \cdot d\vec{A} = E(4\pi r^2) = \frac{Q_{\text{enc}}}{\epsilon_0}
\]
\[Q_{\text{enc}} = \frac{4}{3} \frac{\pi r^3}{\pi r_0^3} Q = \frac{r^3}{r_0^3} Q\]
\[\implies E = \frac{1}{4\pi \epsilon_0} \frac{Q}{r_0^3} r\]

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Use the Symmetry!

Charged Line

\[ \int_{cyl} \vec{E} \cdot d\vec{A} = E(2\pi rl) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0} \]

\[ \Rightarrow E = \frac{1}{2\pi \varepsilon_0} \frac{\lambda}{r} \]

Cylindrical Surface

\[ \vec{E} \perp d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = 0 \]

\[ \vec{E} \parallel d\vec{A} \Rightarrow \vec{E} \cdot d\vec{A} = EdA \]

\[ E(r) = \text{const.} \]
Hollow conducting Sphere

$$\vec{E} = 0 \Rightarrow \oint_{\partial A} \vec{E} \cdot d\vec{A} = 0 = \frac{Q_{\text{encl}}}{\varepsilon_0}$$
Last example

\[ \sigma = \frac{Q}{A} \]

\[ \int_A \vec{E} \cdot d\vec{A} = EA = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \]

\[ \implies E = \frac{\sigma}{\varepsilon_0} \]
Faraday Cage

Hollow Metal Sphere

Van der Graaf Generator

Figure by MIT OCW.
Faraday Cage

Hollow Metal Sphere

Van der Graaf Generator

Large $E; \ E \sim \frac{1}{r^2}$

Figure by MIT OCW.
‘Challenge’ In-Class Demo

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