APPLIED ECONOMICS FOR MANAGERS SESSION 13—

I. REVIEW: EXTERNALITIES AND PUBLIC GOODS
   A. PROBLEM IS ABSENCE OF PROPERTY RIGHTS
   B. REINTRODUCTION OF MARKET/PRICE MECHANISM
   C. PUBLIC GOODS AND TAXATION

II. INFORMATION ISSUES
   A. UNCERTAINTY—HOW DO AGENTS MAKE CHOICES WHEN OUTCOME IS UNCERTAIN?
   B. PUBLIC GOOD ASPECT OF INFORMATION—HOW BEST TO GET INFORMATION PRODUCED GIVEN THAT MC OF SHARING INFORMATION IS ZERO?
   C. ASYMMETRIC INFORMATION—MARKET OUTCOMES WHEN ONE PERSON KNOWS SOMETHING THAT THE OTHER DOESN’T?

III. IMPERFECT INFORMATION: RISK AND UNCERTAINTY
   A. BERNOULLI’S ST. PETERSBURG PARADOX
      1. PAY $1: GET $2^N IF 1ST “HEADS” COMES UP ON NTH TOSS
      2. EXPECTED PAYOFF: \( \frac{1}{2}$2 + \(\frac{1}{4}$4 + \(\frac{1}{8}$8 + \ldots = \infty \)
   B. THE EXPECTED UTILITY HYPOTHESIS
      1. \( V = E(U) = p_1 U(X_1) + p_2 U(X_2) + p_3 U(X_3) + \ldots + p_N U(X_N) \)
      2. RISK AVERSION: \( E(U) < U[E(X)] \) OR, WILLING TO TRADE SOME AMOUNT TO GAIN CERTITUDE
      3. CONCAVITY OF \( U(X) \) AND RISK AVERSION
   C. FORMAL DEFINITION OF CONCAVITY: UTILITY OF EXPECTED VALUE > EXPECTED UTILITY OF POSSIBLE VALUES
      1. EXAMPLE: TWO POSSIBLE OUTCOMES: \( X_0 \) WITH PROBABILITY \( p \), AND \( X_1 \) WITH PROBABILITY \( 1 - p \)
      2. \( U[pX_0 + (1-p)X_1] > pU(X_0) + (1-p)U(X_1) \)
D. CONCAVITY AND RISK AVERSION EXAMPLE: TWO POSSIBLE OUTCOMES: $300,000 WITH PROBABILITY 0.9 AND $60,000 WITH PROBABILITY 0.1

1. AVERAGE OR EXPECTED VALUE = 0.9x$300,000 + 0.1x$60,000 = $276,000 = E(X)

2. AVERAGE OR EXPECTED UTILITY = 0.9xU($300,000) + 0.1xU($60,000) = E[U(X)]

NOTE: DIAGRAM SAYS THAT EXPECTED UTILITY OF GETTING $300,000 WITH PROB = 0.9 AND $60,000 WITH PROBABILITY IS SAME AS UTILITY OF GETTING $260,000 FOR CERTAIN. IN OTHER WORDS, INDIVIDUAL IS WILLING TO GIVE UP $16,000 ON AVERAGE ($276,000 – $260,000) TO AVOID UNCERTAINTY, TO AVOID RISK
IV. UNCERTAINTY AND THE CAPITAL ASSET PRICING MODEL

A. ROUGH INTERPRETATION OF RISK AVERSION: PEOPLE DON’T LIKE VARIABILITY—VARIANCE

1. UNDER SOME CIRCUMSTANCES, WE CAN TRANSLATE THIS INTO A PRECISE UTILITY RELATIONSHIP THAT SAYS PEOPLE GET POSITIVE UTILITY FROM A HIGH AVERAGE EXPECTED WEALTH, \( \bar{W} \), BUT DON’T LIKE VARIANCE, \( \sigma^2 \)

2. FOR INVESTORS, WE CAN EQUIVALENTLY USE EXPECTED RATE OF RETURN, \( E(R) \), AND VARIANCE OF THAT RETURN, \( \sigma^2 \)

B. CONSTANT UTILITY CURVES—MEAN VARIANCE ANALYSIS

\[
\text{E}(R) \quad \sigma^2
\]

C. IMPLICATIONS:

1. RISKIER ASSETS ARE THOSE WHOSE RETURNS HAVE MORE VARIANCE

2. RISKIER ASSETS SHOULD PAY A HIGHER RETURN TO COMPENSATE FOR RISK

3. QUESTIONS:
   a. HOW DO YOU MEASURE VARIANCE, \( \sigma^2 \) OF STOCK’S RETURNS IN AN ECONOMICALLY MEANINGFUL WAY?
   b. WHAT IS THE EXTRA RETURN FOR INCREASES IN \( \sigma^2 \)? I.E, WHAT IS THE PRICE OF RISK?
D. VARIANCE VERSUS COVARIANCE

1. CONSIDER ONE RISKY ASSET WITH VARIANCE $\sigma^2$. IF ONLY THIS ASSET IS HELD, THEN ONE GETS AN EXPECTED RETURN OF $E(R_1)$ AND A VARIANCE OF $\sigma_1^2$.

2. NOW CONSIDER SPLITTING YOUR INVESTMENTS BETWEEN TWO RISKY ASSETS, PUTTING $f_1$ IN ASSET 1 AND $f_2 [= 1 - f_1]$ IN ASSET 2. THEN EXPECTED RETURN IS: $f_1E(R_1) + f_2E(R_2)$ AND THE VARIANCE IS: $f_1^2\sigma_1^2 + f_2^2\sigma_2^2 + 2f_1f_2COV(R_1, R_2)$.

3. NOW CONSIDER SPLITTING YOUR INVESTMENTS THREE WAYS: $f_1$ IN ASSET 1; $f_2$ IN ASSET 2; AND $f_3 [= 1 - f_1 - f_2]$ IN ASSET 3
   a. EXPECTED RETURN IS: $f_1E(R_1) + f_2E(R_2) + f_3E(R_3)$.
   b. VARIANCE IS
      \[ f_1^2\sigma_1^2 + f_2^2\sigma_2^2 + f_3^2\sigma_3^2 + 2f_1f_2COV(R_1, R_2) + 2f_1f_3COV(R_1, R_3) + 2f_2f_3COV(R_2, R_3) \]

4. IN THE CASE OF FOUR ASSETS THE RESULTS ARE:
   a. $E(R) = f_1E(R_1) + f_2E(R_2) + f_3E(R_3) + f_4E(R_4)$.
   b. $VAR = f_1^2\sigma_1^2 + f_2^2\sigma_2^2 + f_3^2\sigma_3^2 + f_4^2\sigma_4^2$
      \[ + 2f_1f_2COV(R_1, R_2) + 2f_1f_3COV(R_1, R_3) + 2f_1f_4COV(R_1, R_4) \]
      \[ + 2f_2f_3COV(R_2, R_3) + 2f_2f_4COV(R_2, R_4) + 2f_3f_4COV(R_3, R_4) \]

5. MORAL: AS ONE DIVERSIFIES MORE, WHAT MATTERS ABOUT ANY ONE ASSET IS NOT THE VARIANCE OF ITS RETURNS BY ITSELF, BUT THE COVARIANCE OF ITS RETURNS WITH THE RETURNS OF ALL THE OTHER ASSETS.

E. DEFINE A VERY WELL-DIVERSIFIED BUNDLE OF ASSETS AS "THE MARKET BUNDLE"

1. THIS BUNDLE HAS AN EXPECTED RETURN $\bar{R}_M$ AND
2. STANDARD DEVIATION OF THAT RETURN $= \sigma_M$. 

3. DEFINE THE “RISK” IN ANY ASSET OR COLLECTION OF ASSETS AS A MEASURE OF ITS COVARIANCE WITH THE RETURNS ON THE MARKET BUNDLE. CALL THIS TERM THE ASSET BETA, I.E., \( \beta_1 \) FOR ASSET 1.


5. NOW IDENTIFY THE RISK-FREE RATE, \( R_F \).

6. IF THE MARKET BUNDLE HAS AN EXPECTED RETURN OF \( \bar{R}_M \) AND SINCE IT HAS ONE UNIT OF RISK (A BETA) OF 1, THE PRICE PER UNIT OF RISK IS \( \bar{R}_M - R_F \).

7. FOR ANY OTHER ASSET, WE MEASURE ITS BETA AS A MEASURE OF ITS RISK, AND THEN SAY THAT IT MUST PAY A RISK PREMIUM OF ITS BETA TIMES THE PRICE OF RISK, E.G. FOR ASSET A, THE PRICE MUST BE SUCH THAT ITS EXPECTED RETURN SATISFIES:

\[
E(R_A) = R_F + \beta_A(\bar{R}_M - R_F)
\]

F. THE FOREGOING EQUATION IS KNOWN AS THE SECURITY MARKET LINE. IT IS ONE OF THE MOST FUNDAMENTAL EQUATIONS IN FINANCIAL THEORY. WHILE ITS LITERAL ACCURACY IS DEBATABLE IT MAKES THREE, BROAD POINTS.

1. PEOPLE ARE RISK AVERSE AND NEED TO BE COMPENSATED FOR BEARING RISK

2. THE APPROPRIATE MEASURE OF RISK IS COVARIANCE. YOU WON’T GET COMPENSATED FOR TAKING RISK THAT YOU COULD COSTLESSLY AVOID.

3. “BEATING THE MARKET RETURN \( \bar{R}_M \) IS EASY.” JUST INVEST IN ASSETS WITH BETAS GREATER THAN ONE. THE REAL QUESTION IS DID YOU GET A HIGHER RETURN AFTER ADJUSTING FOR RISK?