Game Theory
for
Strategic Advantage

15.025

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Part III: “Big” Applications

- Repetition
  - Long-Run Relationships
  - Classes 12-14

- Asymmetric Information
  - Auctions & Market Design
  - Classes 15-18

- Signaling
  - Credibility & Reputation
  - Classes 19-21

Prof. Alessandro Bonatti
Digital Divide

Google dominates the global online-ad market. Top five companies, by net online-advertising revenue in billions:

- **Google**: $38.32 billion (2013), $32.73 billion (2012)
- **Facebook**: $6.99 billion
- **Microsoft**: $3.46 billion, $3.07 billion
- **Yahoo**: $3.44 billion, $3.51 billion
- **IAC**: $1.53 billion, $1.40 billion

Source: Company reports via eMarketer, The Wall Street Journal
Uncertainty Example: an Auction

• Two firms (GE vs. W) bid for a contract.
• The value of the contract to GE is \( v_{GE} = $65M \).
• Say you are GE: how much do you bid?
• Do you have all the information you’d like?
• GE doesn’t know \( v_{W} \).
• W doesn’t know \( v_{GE} \).
Today’s Class

1. Uncertainty in games
2. New equilibrium notion
3. Applications: basic auctions

Looking Ahead

1. Reserve prices & winners’ curse
2. Online auctions
3. Designing auctions and markets
Uncertainty in Canonical Games

**Game Type**
- Prisoners’ Dilemma
- Chicken / Entry
- Stag Hunt
- ...
- Coordination
- Beauty contest

**Source of Uncertainty**
- Gain from defection
- Cost of acting tough / entry
- Go-it-alone value
- ...
- Strength of common interest
- Opponents’ sophistication

What game is my opponent seeing?
Our Old Entry Game

- The (gross) value of winning the market alone is 50.
- Each player $i=\{1,2\}$ has a cost 30 of investing.
- If both enter, price competition erases all (gross) profits

<table>
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<th>In</th>
<th>Out</th>
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<tbody>
<tr>
<td><strong>In</strong></td>
<td>(-30, -30)</td>
<td>(20, 0)</td>
</tr>
<tr>
<td><strong>Out</strong></td>
<td>(0, 20)</td>
<td>(0, 0)</td>
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- No dominated strategies
Entry Game Revisited

• The (gross) value of winning the market alone is 50.
• Each player $i=\{1,2\}$ has a cost $c_i$ of entering.
• If both enter, price competition erases all (gross) profits

Player 2

Player 1

<table>
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<th>Out</th>
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<tbody>
<tr>
<td>In</td>
<td>(-$c_1,-c_2$)</td>
<td>(50-$c_1,0$)</td>
</tr>
<tr>
<td>Out</td>
<td>(0,50-$c_2$)</td>
<td>(0,0)</td>
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</tbody>
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• Any dominated strategies?
• What if I’m not sure about Pl. 2’s cost?
Each player’s $c_i$ is uniformly drawn from $[0, 100]$. The two draws are independent. Players know their own cost only.

• How to proceed? Let’s play!!
Expected Payoffs ($c_1 > 50$)

Player 1’s expected payoff from IN:

$$E[\text{payoff from "IN"}] = -c_1 + 50(1 - p_2)$$

Player 1 should stay OUT if $c_1 > 50$

$p_2 =$ Player 2’s prob. of IN (from 1’s point of view)
Expected Payoffs ($c_1 < 50$)

Player 1’s expected payoff from IN:

\[ E[\text{payoff from “IN”}] = -c_1 + 50*(1 - p_2) \]

Player 1 should enter if $c_1 < 50*(1 - p_2)$

Diagram:
- Y-axis: $50 - c_1$
- X-axis: $-c_1$
- Line from (0, 50 - $c_1$) to (1, 0)

$p_2 =$ Player 2’s prob. of IN

Player 1’s expected payoff from OUT: $E[\text{payoff from OUT}] = 0$
For which cost levels does Pl. 2 choose IN?

- Suppose player 2 chooses IN if $c_2 < 50$.
- Then $p_2 = \text{Prob (IN)} = \text{Prob (} c_2 < 50 \text{)} = 0.5$,
- Then Pl. 1 $\rightarrow$ IN if $c_1 < 25$. Which means $p_1 = 0.25$
- But then Pl. 2 should go IN if and only if $c_2 < 37.5$.
- ... which means Pl. 1 $\rightarrow$ IN if $c_1 < 31.25$.

More general criterion: Reaction Functions

$$c_1 = 50(1-p_2) = 50(1-c_2/100) = 50 - c_2/2$$
"Reaction Functions"

Player 1’s maximum

Player 2’s maximum

\[ c_2^* = 50 - \frac{c_1^*}{2} \]

\[ c_1^* = 50 - \frac{c_2^*}{2} \]

\( (33.3, 33.3) \) Nash Equilibrium
Solving for Equilibrium

- Equilibrium = two cut-offs \((c_1^*, c_2^*)\) such that
  - \(c_1^* = \text{max}(c_1) \rightarrow \text{IN given that Pl. 2} \rightarrow \text{IN if } c_2 < c_2^*\)
  - \(c_2^* = \text{max}(c_2) \rightarrow \text{IN given that Pl. 1} \rightarrow \text{IN if } c_1 < c_1^*\)

- \(c_1^* (c_2^*) = 50 - \frac{c_2^*}{2}\) and \(c_2^* (c_1^*) = 50 - \frac{c_1^*}{2}\)
- \(c_1^* = c_2^* = 100/3 = 33.3\ldots\)
- \(p_1 = p_2 = 1/3\)
- \(E[\text{payoff(IN)}] = -c_i + 50*(1-1/3) = 33.3 - c_i\)
A (Bayesian) Nash Equilibrium

• A Nash equilibrium of this (Bayesian) game is:

  1) A critical value \( c_1 \) for Pl. 1 such that playing \( IN \) for costs below \( c_1 \) is a best response to Pl. 2’s play

  2) A critical value \( c_2 \) for Pl. 2 such that playing \( IN \) for costs below \( c_2 \) is a best response to Pl. 1’s play

• Best response = maximize expected payoff!
Right and Wrong Information

• In the BNE, entry is profitable only if $c<33.3$
• Cost distribution: uniform $[0, 100]$
• Expected cost = 50
• On average, my opponent’s dominant strategy is \textbf{OUT}
• Best response to expected cost = \textbf{IN}!! (given $c<50$)

• This uses the \textbf{wrong information}!! (expected cost)
• \textbf{Right information: expected action} (IN with Pr=1/3)
• \textbf{Correct strategy: IN if } $c<33.3$