Game Theory for Strategic Advantage

15.025

Alessandro Bonatti
What Have We Learned So Far?

- You must account for your own and your opponents’ rationality / sophistication

- In some games, it is appropriate (useful!) to exploit the logic of rationalizability

- Having the right game in mind is a source of competitive advantage. (Recall Epson vs. HP)
1. Building a Language: *ask lots of questions!*

2. Nash Equilibrium: *the Good, the Bad...*

3. Prototypical Games: *PD, Coordination, Chicken*
A Structured Approach

1) Game theory is a **toolkit for strategic analysis**

2) Specify a game: payoffs represent **total utility**

3) Use **all available information** to describe the game

4) But once we are in the game, **we are in the game**.

5) Base the **analysis** on the game’s elements alone
More formally ...

A **game** is a **multi-player decision problem**:

- **players** \( i = 1, 2, \ldots, n \)
- **strategies** \( a_i \) from “feasible set” \( A_i \)
- **payoff functions** (utilities) \( u_1(a_1, a_2), u_2(a_1, a_2) \)

- **typically:**
  - \( A_{\text{Row}} = \{\text{Top, Bottom}\} \)
  - \( A_{\text{Column}} = \{\text{Left, Right}\} \)
  - **Payoff** = total utility in “payoff matrix”

- **many more examples** (like beauty contest)
Dominated Strategies

Definition: Strategy $x$ is *dominated* for player $i$ if there exists another strategy $y$ that guarantees a higher payoff to player $i$.

For example:

<table>
<thead>
<tr>
<th></th>
<th>No Ad</th>
<th>Ad</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No Ad</strong></td>
<td>50, 50</td>
<td>20, 60</td>
</tr>
<tr>
<td><strong>Ad</strong></td>
<td>60, 20</td>
<td>30, 30</td>
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</tbody>
</table>

No Ad is dominated by Ad (for both players)
Dominant Strategies

A **dominant strategy** for player *i* **always** gives player *i* a **higher payoff** than any other strategy.

**For example:** placing yourself at the **median voter’s** location (if all you care about is winning the election)
Reynolds’ best strategy is Ad regardless of what Philip Morris does

Ad is a “dominant strategy”
Dominance: a Risk-Free Concept

PENALTY KICKS

<table>
<thead>
<tr>
<th>Kicker</th>
<th>Goalie</th>
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<tbody>
<tr>
<td></td>
<td>Left</td>
</tr>
<tr>
<td>Left</td>
<td>2, 5</td>
</tr>
<tr>
<td>Middle</td>
<td>3, 3</td>
</tr>
<tr>
<td>Right</td>
<td>5, 1</td>
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</table>

Middle is not dominated for the Kicker!
(but it’s not a great idea either)
Iterative Elimination of *Strictly* Dominated Strategies

*Strategies that survive all rounds of elimination are called rationalizable strategies*

Required assumptions:

- Know the game
- Rational player
- Rational opponents
- Knowledge of knowledge of ... of rationality

Order of elimination does **not** matter
Example: Tourists & Natives

- Two bars can charge a price per drink of $2, $4, or $5
  - 6,000 tourists pick a bar randomly
  - 4,000 natives select bar with lowest price

- Example: Both charge $2
  - each gets 5,000 customers ➔ payoff = $10,000

- Example: Bar 1 charges $4, Bar 2 charges $5
  - Bar 1 gets 3,000 + 4,000 = 7,000 customers (➔ $28,000)
  - Bar 2 gets 3,000 customers (➔ $15,000)
For each Bar, $2 is dominated by both $4 and $5

In the reduced game (with only 4 and 5), $5 is dominated by $4

($4,$4) is the only rationalizable strategy profile
### Traffic Game

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
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<tbody>
<tr>
<td></td>
<td>Drive</td>
<td>Stop</td>
</tr>
<tr>
<td>Row</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drive</td>
<td>(-2, -2)</td>
<td>(1, 0)</td>
</tr>
<tr>
<td>Stop</td>
<td>(0, 1)</td>
<td>(-1, -1)</td>
</tr>
</tbody>
</table>

**Rationalizable Outcomes = all four!!**
Beliefs and Best Responses

Definition: Player $i$’s belief about the strategy that $i$’s opponents will play is a probability distribution over their actions.

Definition: Strategy $x$ for player $i$ is a best response if $x$ maximizes $i$’s expected payoff, given $i$’s beliefs.

Example: In the traffic game, “drive” is a best response if player 1 believes player 2 “stops” with probability $>50\%$. 

Best Responses: more advanced examples

• “Guess 0.75*Average” ➔ best response = 75% of your estimate of your opponents’ average.

• Hide and Seek ➔ best response = hide where you think your opponents are least likely to search, seek where you think they are most likely to hide.

• Product Positioning ➔ best response = locate to the left/right of your competitor’s expected position***
Nash Equilibrium

**Definition:** A profile of strategies (i.e., one for each player) is a *Nash Equilibrium* if each player’s strategy is a *best response* to the other players’ strategies.

**Examples:**
- both firms locating their product at the center of the line;
- everyone choosing the number 1;
- player 1 driving and player 2 stopping.
In the movie

Two pharma firms choose which compound to pursue...

Two friends at a bar...

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<thead>
<tr>
<th></th>
<th>Pfizer</th>
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<tbody>
<tr>
<td>Merck</td>
<td>P1</td>
<td>P2</td>
<td>P3</td>
</tr>
<tr>
<td>P1</td>
<td>(0,0)</td>
<td>(5,1)</td>
<td>(5,1)</td>
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<tr>
<td>P2</td>
<td>(1,5)</td>
<td>(0,0)</td>
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<tr>
<td>P3</td>
<td>(1,5)</td>
<td>(1,1)</td>
<td>(0,0)</td>
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($4, $4) is the only **rationalizable** strategy profile

($4, $4) is also the unique **nash equilibrium**

Iterated elimination of dominated strategies **might** **yield** a Nash equilibrium and **cannot eliminate** one.
Nash **Equilibrium**: the Good

- It always exists (John Nash, 1950)
- Easy to find
  - For us
  - For firms (given enough time)
- It is “stable”
- A tool for out-of-sample predictions
- A criterion for investment decisions (next class)
  - What if demand ↗? ↘?
  - What if one firm cuts its costs?
Nash Equilibrium: the Bad

- Equilibrium does not mean optimal!
- Think of the prisoners’ dilemma:
- Unique Nash equilibrium, but “Pareto-inefficient”
- Many interesting games have >1 Nash Equilibrium!
- Stability not-so-great anymore!
Nash Equilibrium: the Many

No dominated strategy for either player

Two Pareto-ranked Nash equilibria

Could have path-dependence!

Coordination Game

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<td>L</td>
</tr>
<tr>
<td>T</td>
<td>(2, 2)</td>
</tr>
<tr>
<td>B</td>
<td>(0, 0)</td>
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Selecting Nash Equilibrium

• How to “steer the game”?  

• Commitment tactics! (Cigarettes, ice-cream vendors)

• Before then…. Do we have the right game?
## The Last Chocolate

<table>
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<tbody>
<tr>
<td><strong>Row</strong></td>
<td></td>
</tr>
<tr>
<td>Split</td>
<td>Split (2, 2)</td>
</tr>
<tr>
<td>Steal</td>
<td>(4, 0)</td>
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- **Dominant strategy for each player**
- **Seemingly self-evident solution to the game**
Tiny Details Matter

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<td>Row</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>(5, 5)</td>
</tr>
<tr>
<td>B</td>
<td>(10, 0)</td>
</tr>
</tbody>
</table>

• Is B a dominant strategy for Row?
• Is R a dominant strategy for Column?
• Find all Nash Equilibria

Youtube ➔ “Best Split or Steal Ever”
Stag Hunt

(J.-J. Rousseau, 1754)

- Hunting stag vs. rabbit
- No dominated strategy for either player
- Two Pareto-ranked Nash equilibria
- The role of trust

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<tr>
<td></td>
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<tr>
<td>T</td>
<td>(1, 1)</td>
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Example: Technology Adoption

One upstream and one downstream firm

- Currently **old** inventory-management system \(\rightarrow\) profit = $1M each
- Each can independently invest $1M in upgrading to a **new** system
- If only one upgrades, no one benefits
- If both upgrade, they exploit synergies worth $3M (gross)

What’s the game?

- Dominated strategies? **No**
- What’s the Prediction?

Two Nash Equilibria:

- \((Old, Old)\) and \((New, New)\)
- This is a coordination game that may explain some interesting real world phenomena (mergers to coordinate on synergies)
Takeaways: Building a Language

1. Dominant Strategy: *performs better than all other strategies, regardless of opponents’ behavior*

2. Dominated Strategy: *an alternative strategy always performs better, regardless of opponents’ behavior*

3. Rationalizable Strategies: *survive the iterated elimination of dominated strategies*

4. Best-Responses

5. Nash Equilibrium
Building a New Approach

1) Once in the game...

2) Assumptions → Conclusions

3) Do we like the conclusions?

- Can we trust the assumptions?
- Were the assumptions wrong? Or did we just learn something?
- Who is the opponent?
- What is the *actual* game?